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AN APPROACH TO COMPARING NUMBER MODULES IN NUMBER SYSTEMS IN RESIDUAL

Alexey AlexeevichLyubomudrov[1]

[1]National Research Nuclear University «MEPhI», 31, Kashirskoe highway, Moscow, 115409, Russia

Abstract: The paper considers an approach to comparing the number modules represented in the number systems in residual classes (RNS), one of the bases of which is $p_n = 2^k$, where k = 2, 3, 4, ... The approach involves the following sequence of actions.

The decrease in number modules |A| and |B| by α_n and β_n, respectively, where α_n = rest|A| mod p_n and β_n = rest|B| mod p_n. Next, access to the computer memory at the addresses ($\alpha_1, \alpha_2, \ldots, \alpha_{n-1}$) and ($\beta_1, \beta_2, \ldots, \beta_{n-1}$) and selection from the memory the high digits (without k low digits) of the modules |A| and |B| represented in the positional binary system by comparing the selected high digits of the modules. In this case, a larger module will correspond to larger high digits. If the high digits of the modules are equal then the lower digits are compared which coincide with the residues α_n and β_n. In this case, the largest of the lower digits will correspond to the larger module. With this approach, the memory required to store the compared modules when they are written in the positional binary number system is reduced by 2^k times, and the word length of the stored words decreases by k binary digits. In addition, the low bit depth of the RNS bases allows the using of tabular calculation methods which increases the speed of calculations.

Thus the proposed approach has a practical orientation and may be of interest to computer developers.

Keywords: number systems in residual classes; number modules; comparison of modules; positional number systems; choice of base values; tabular calculations.

I. INTRODUCTION

The use of number systems in residual classes (RNS) allows parallelizing the calculation process at the level of arithmetic operations which significantly increases the speed of computers. In view of this, a considerable attention is paid to the organization of computing processes using RNS [1-8]. One of the difficulties in applying RNS is a comparison of the number values and in particular of the number modules. Accordingly the purpose of this article is to consider the proposed approach to comparing the number modules which are presented in the RNS. Due to the fact that the choice of values of the bases of the RNC has a noticeable effect on the organization of the computational process [9-11], we will assume that one of the bases of the RNC is $p = 2^k$, where k = 2, 3, 4, ... For example the presence of a base with such a value simplifies in particular the implementation of number conversions from the positional number systems to RNS and vice versa [10-11].

The construction of an effective method for comparing of number modules will create the foundation for the formation of signs of results when performing the operations of algebraic addition in RNS.

II. PROBLEM RESOLUTION

Let the values of the bases of the RNS are equal to $p_1, p_2, \ldots, p_n$ and the value of one of the bases is $p_n = 2^k$ where k = 2, 3, 4, ... Suppose that there are two integers A and B whose modules when represented in the RNS are respectively equal to |A| = ($\alpha_1, \alpha_2, \ldots, \alpha_n$) and |B|= ($\beta_1, \beta_2, \ldots, \beta_n$) where αi = rest A mod pi and βi = rest B mod pi, i = 1, 2, ..., n.

It is required to carry out a comparison of |A| and |B| presented in the RNS.
To perform the comparison the following approach is possible which involves the following actions:

1. Decreasing the modules |A| and |B| by the quantities α_n = rest A mod p_n and βn = rest B mod p_n. This achieves an integer division of |A|- α_n and |B| - β_n by $p_n = 2^k$ and the appearance of k zeros in the lower of k digits when |A|- α_n and |B| - β_n are represented in the positional binary system .
The decreasing of modules is performed in the RNS by doing the following operations:
$|A|- \alpha_n =$
$= \{(\alpha_1 - \alpha_n) \bmod p_1, (\alpha_2 - \alpha_n) \bmod p_2, \ldots, (\alpha_{n-1} - \alpha_n) \bmod p_{n-1}, (\alpha_n - \alpha_n) \bmod p_n\}=$
$= \{(\alpha_1 - \alpha_n) \bmod p_1, (\alpha_2 - \alpha_n) \bmod p_2, \ldots, (\alpha_{n-1} - \alpha_n) \bmod p_{n-1}, 0\}$;
$|B| - \beta_n =$
$= \{(\beta_1 - \beta_n) \bmod p_1, (\beta_2 - \beta_n) \bmod p_2, \ldots, (\beta_{n-1} - \beta_n) \bmod p_{n-1}, (\beta_n - \beta_n) \bmod p_n\}=$
$= \{(\beta_1 - \beta_n) \bmod p_1, (\beta_2 - \beta_n) \bmod p_2, \ldots, (\beta_{n-1} - \beta_n) \bmod p_{n-1}, 0\}$.
When performing these operations it is advisable to represent residues in a binary number system.

2. By the residues $\{(\alpha_1 - \alpha_n) \bmod p_1, (\alpha_2 - \alpha_n) \bmod p_2, \ldots, (\alpha_{n-1} - \alpha_n) \bmod p_{n-1}\}$ and $\{(\beta_1 - \beta_n) \bmod p_1, (\beta_2 - \beta_n) \bmod p_2, \ldots, (\beta_{n-1} - \beta_n) \bmod p_{n-1}\}$ like binary addresses the computer memory is being accessed and the highest binary digits (without k lower digits) of the numbers |A| - α_1 and |B| - β_1 are selected from the memory. The selection from the computer memory of the lower digits is not required since they are known and are equal to zeros.

3. A comparison of the higher binary digits of the numbers |A| - $\alpha_1$aled|B| - β_1 is made. This digits are selected from the computer memory without taking into account the k lower digits which are equal to zeros. Based on the comparison results the ratio of the quantities |A| and |B| is determined. Here the largest module will correspond to the higher value of the higher binary digits. This is due to the fact that the binary representations of the numbers |A| and |A| - α_1 as well as the binary representations of the numbers |B|aled|B| - β_1 have differences only in the records of k low binary digits.

4. If the higher digits of the numbers |A| - α_1 and |B| - β_1 are equal then an additional comparison of the k low binary digits is performed which are equal to the residues α_n = rest A mod p_n and β_n = rest B mod p_n. In this case a larger module|A| or |B| will correspond to a larger residue α_n or β_n.

The implementation of the approach is illustrated by the following example.
Let the modules of numbers A and B in decimal and binary number systems have the following representations:
$|A| = 328_{10} = 101001000_2$, $|B| = 415_{10} = 110011111_2$.
It is required to carry out a comparison of the modules of these numbers using the considered approach when the modules are initially represented in an RNS with three bases $p_1 = 13$, $p_2 = 15$, $p_3 = 2^{k=4} = 16$.
We apply the considered approach.

When writing in RNS with the indicated bases the number modules will have the following form::

$|A| = (\alpha_1, \alpha_2, \alpha_3) = (3, 13, 8)$ where α_1 = rest 328_{10} mod 13, α_2 = rest 328_{10} mod15, α_3 = rest 328_{10} mod 16.
$|B| = (\beta_1, \beta_2, \beta_3) = (12, 10, 15)$ where β_1 = rest 415_{10} mod 13,
β_2 = rest 415_{10} mod 15,β_3 = rest 415_{10} mod 16.

Then in accordance with the considered approach the following sequence of actions is performed:

1. Subtractions in RNS from |A| and |B| the residues $\alpha_3 = 8$ and $\beta_3 = 15$ respectively:
$|A| - \alpha_3 = (3, 13, 8)_{13, 15, 16} – (8, 8, 8)_{13, 15, 16} = (8, 5, 0)_{13, 15, 16} = (1000_2, 0101_2, 0000_2)_{13, 15, 16}$;
$|B| - \beta_3 = (12, 10, 15)_{13, 15, 16} – (15, 15, 15)_{13, 15, 16} = (10, 10, 0)_{13, 15, 16} = (1010_2, 1010_2, 0000_2)_{13, 15, 16}$.
Subtraction is accompanied by zeroing of the k = 4 low digits in the modules |A| and |B| when they are written in the positional binary system.

2. Access to the computer memory at the addresses 1000 0101_2 and 10101010_2 respectively and the selection from the computer memory high binary digits of the numbers |A| - α_3 and |B| - β_3 without k low binary digits which are equal to zeros.
The selection of the high digits of the numbers |A| - $\alpha_3$aled|B| - β_3 is sufficient since this digits are equal to the high digits of the modules |A| and |B|.
Indeed $|A| = 328_{10} = 101001000_2$ and accordingly $|A| - \alpha_3 = 320_{10} = 101000000_2$. Also $|B| = 110011111_2$ and accordingly $|B| - \beta_3 = 400_{10} = 1100100000_2$.

3. The high binary digits of the numbers |A| - α_3 and |B| - β_3 are compared, i.e. the digits 10100_2 from |A| - α_3 are compared with the digits 11001_2 from |B| - β_3. Since $11001_2 > 10100_2$, then |B|>|A|.

4. If the high binary digits of the numbers |A| - $\alpha_3$aled|B| - β_3 were equal then in this case we would additionally compare the residues $\alpha_3 = 8_{10} = 1000_2$ and $\beta_3 = 15_{10} = 1111_2$. And in this case a larger residue would correspond a larger module.

The high efficiency of the considered approach to the comparing of number modules is due to the fact that all operations when comparing the modules presented in the RNS are conveniently to do with a table calculations for example by accessing to the LSI memory. This is due to the low bit depth of the residues.

III. Results and Discussion

The result of the work in the proposed approach to comparing the number modules represented in the RNS. One of the main features of the approach is that one of the bases of the RNS is chosen equal to $p_n = 2^k$, where k = 2, 3, 4,. . . .

If the bases of the RNS are respectively equal to $p_1, p_2, . . . , p_n$ then in this system there are $N = p_1 \cdot p_2 \cdot, . . . , \cdot p_n$ number modules. For direct comparison of the mentioned modules it is necessary to store $N = N = p_1 \cdot p_2 \cdot, . . . , \cdot p_n$ of records of modules in a positional binary number system in the computer memory where the length of records m will be $m \geq \log_2 N$.

The considered approach assumes that one of the bases of the RNS is chosen equal to $p_n = 2^k$, where k = 2, 3, 4,. . . . This allows to reduce the quantity of modules stored in memory by $p_n = 2^k$ times and to reduce the bit depth of words stored in memory by k binary digits.

In addition the low bit depth of the bases of the RNC creates convenience for the using of tabular calculations in which the results are stored in the computer memory and the addresses are operands. This improves the performance of devices created with the help of the above approach.

Because of this the considered approach to the comparing of the number modules has a practical orientation and is of interest to developers of computer technology.

IV. Conclusion

The paper considers an approach to comparing number modules in RNS one of the bases of which is $p_n = 2^k$, where k = 2, 3, 4,. . . .

This approach provides the decreasing of the number modules |A|aled|B| by the values $\alpha_n$aledβ_n respectively which is performed in the RNS, the selection of the high order digits of the compared modules from the computer memory in a binary and their subsequent comparison. If it turns out that the high order digits of the modules are equal then the additionally comparing of the lower k binary digits is done which coincide with α_n = rest |A| mod p_n and β_n = rest |B| mod p_n respectively.

With this approach the memory required to store the compared number modules when they are written in a binary is reduced. The number of stored words is reduced by 2^k times and bit depth of words is reduced by k binary digits.

In addition the low bit depth of the bases of the RNS allows the using of tabular calculations which increases the speed of devices.

Thus the proposed approach has a practical orientation and may be of interest to computer developers.

V. References

[1] Wang, Y., Aboulhamid, M.and Shen, H. 2002. Adder based residue to binary numbers converters for (2n–1, 2n, 2n+1). IEEE Trans.Signal Processing, 50(7): 1772–1779.

[2] Tomczak, T. 2008. Fast sign detection for rns (2n–1,2n, 2n+1). IEEE Trans. on Circuits and Systems-I: Regular papers, 55(6): 1502–1511.

[3] Chervyakov, N. I. and Averbukh, V. M. 2012. Approximated method of non-modular procedures in RNS. Basic researches, 6: 189 – 193.

[4] Mytsko, E. A. and Mal'chukov, A. N. 2013. Study into hardware implementation of spreadsheet and matrix algorithm of CRC 32 computing. Izv. Tomsk Polytechnic University, 322(5): 182 – 186.

[5] Anikueva, O. V., Lyalhov, P. A. and Chervyakov. N.I. 2014. Implementation of discrete wavelet transformation in RNS of special type. Infokommunikatsionnye Tekhnologii, 12(4): 4 – 9.

[6] Magomedov, Sh. G. (2014) Transformation of number presentation in modular arithmetic in RNS with different bases. Vestn. Astrakh. State Technical University. Series: Management and IT, 4: 32 - 39.

[7] Samoilenko, D. V., Evdokimov, A. A., Koldaev, A.I., et al. 2014. Identification of ACS subjects in RNS. Informational and communication technologies in science, industry and education (Infokom - 6): Proceedings of 6th International R&D conference, Stavropol, April 21 – 27, 2014. Part 2. – Stavropol, pp. 395 – 398.

[8] Knyaz`kov, V.S. and Isupov, K.S. 2015. Device for determination of numbers signs in system of remainder classes. RF Patent, No. 2557446.

[9] Lyubomudrov, A. A. 2011. Device for conversion of binary code into RNS code. RF Patent, No. 2413279.

[10] Lyubomudrov, A. A.and Zaitsev, A. V. 2014. Method of number conversion from positional number system into residual number system. Vestn. MEPhI, 3 (2): 252 – 253.

[11] Lyubomudrov, A. A.andBashkov, A.A. 2016. On Some Problems and Approach to Solution Thereof upon Computing in Residue Number System. Journal of Theoretical and Applied Informational Technology, 86: 377 - 381.

[12] Chervyakov, N. I., Babenko, M. G., Lyakhov, P.A., et al. 2014. Device for comparison of number represented in RNS. RF Patent, No. 2503992.

[13] Isupov, K. S. 2014. On an algorithm of number comparison in RNS. Vestn. Astrakh. State Technical University. Series: Management and IT, 3: 40 – 49.

[14] Knyaz`kov, V. S.andOsinin, I. P. 2013. Method of multiplication arrangement of floating point numbers presented in RNS. RF Patent, No. 2485574.

[15] Pchel`nik, V. K.andRevchuk, I. N. 2012. Implementation of Gauss method in MS Excel spreadsheets. Proceedings of International R&D conference: Informational support of engineering equipment,Moscow, April 10-11, 2012. Moscow, pp. 228 – 229.