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DEMPSTER-SHAFER THEORY BY PROBABILISTIC REASONING

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Abstract: Probabilistic reasoning is used when outcomes are unpredictable. We examine the methods which use probabilistic representations for all knowledge and which reason by propagating the uncertainties can arise from evidence and assertions to conclusions. The uncertainties can arise from an inability to predict outcomes due to unreliable, vague, in complete or inconsistent knowledge. Some approaches taken in Artificial Intelligence system to deal with reasoning under similar types of uncertain conditions.

Keywords: Probability, Probabilistic Reasoning, Dempster-Shafer theory, Belief function, Plausibility, Mutually exclusive, Lattice.

I. Introduction

The probability of an uncertain event A is a measure of the degree of likelihood of occurrence of the event. The set of all possible events is called the sample space S. A probability measure is a function P(·) which maps event outcomes $E_1, E_2, \ldots$, from S onto real numbers and which satisfies the following axioms of probability:

1. $0 \leq P(A) \leq 1$ for any event $A \subseteq S$.
2. P(S) = 1, a certain outcome.
3. For $E_i \cap E_j = \emptyset$, for all $i \neq j$ (E_i are mutually exclusive). $P(E_1 \cup E_2 \cup E_3 \cup \ldots) = P(E_1) + P(E_2) + P(E_3) + \ldots$

The axioms are not sufficient to compute the probability of an outcome. This requires an understanding of the underlying distributions which must be established through one of the following approaches:

1. Use of a theoretical argument which accurately characterizes the processes.
2. Using one's familiarity and understanding of the basic processes to assign subjective probabilities, or
3. Collecting experimental data from which statistical estimates of the underlying distributions can be made.

The Dempster-Shafer theory is based on the notion that separate probability masses may be assigned total subsets of a universe of discourse rather than just to indivisible single members as required in traditional probability theory. It permits the inequality $P(A) + P(\tilde{}A) \leq 1$.

II. Dempster – Shafer Theory

We assume a universe of discourse U and a set corresponding to n propositions, exactly one which is true. The propositions are assumed to be exhaustive and mutually exclusive. Let 2^U denote all subsets of U including the empty set and U itself (there are 2^n such subsets). Let the set function m (sometimes called a basic probability assignment) defined on 2^U, be a mapping to [0,1],

$$m : 2^U \rightarrow [0,1], \text{ be such that for all subsets } A \subseteq U$$

$$m(\emptyset) = 0$$

$$\sum_{A \subseteq U} m(A) = 1$$

The function m defines a probability distribution on 2^U. It represents the measure of belief committed exactly to A. A belief function, Bel, corresponding to a specific m for the set A, is defined as the sum of beliefs committed to every subset of A by m. That is Bel(A) is a measure of the total support or belief committed to the set A and sets a minimum value for its likelihood. It is defined in terms of all belief assigned to a as well as to all proper subsets of A. Thus,

$$\text{Bel}(A) = \sum_{B \subseteq A} m(B)$$

Example: If U contains the mutually exclusive subsets A,B,C, and D then

$$\text{Bel}(\{A,C,D\}) = m(\{A,C,D\}) + m(\{A,C\}) + m(\{C,D\}) + m(\{A\}) + m(\{c\}) + m(\{D\}).$$

In Dempster-Shafer theory, a belief interval also be a subset A. It is represented as the subinterval [Bel(A), PI(A)] of [0,1]. Bel(A) is also called the support of A and PI(A) = 1 - Bel(˜A), the plausibility of A.

$$PI(A) \geq Bel(A),$$

$$Bel(A) + Bel(\tilde{}A) \leq 1,$$

$$PI(A) + PI(\tilde{}A) \geq 1, \text{ and}$$

For $A \subseteq B$,

$$Bel(A) \leq Bel(B),\ PI(A) \leq PI(B).$$

A few specific interval belief values will help to clarify the intended semantics. For example,

[0,1] represents no belief in support of the proposition

[0,0] represents the belief the proposition is false

[1,1] represents the belief the proposition is true

[.3,1] represents partial belief in the proposition

[0,.8] represents partial disbelief in the proposition

[.2,.7] represents belief from evidence both for and against the proposition.

When evidence is available from two or more independent knowledge sources Bel1 and Bel2, one would like to pool the evidence to reduce the uncertainty. Such a combining unction denoted as Bel_1 o Bel_2.

Let two basic probability assignment functions, m_1 and m_2 corresponding to the belief functions Bel_1 and Bel_2, let $A_1,\ldots\ldots\ldots A_k$ be the focal elements for Bel_1 and $B_1,\ldots\ldots\ldots\ldots,B_p$ be the focal elements for Bel_2. Then $m_1(A_i)$ and $m_2(B_j)$ each assign probability masses on the unit interval. They can be orthogonally combined as depicted with the square in figure.

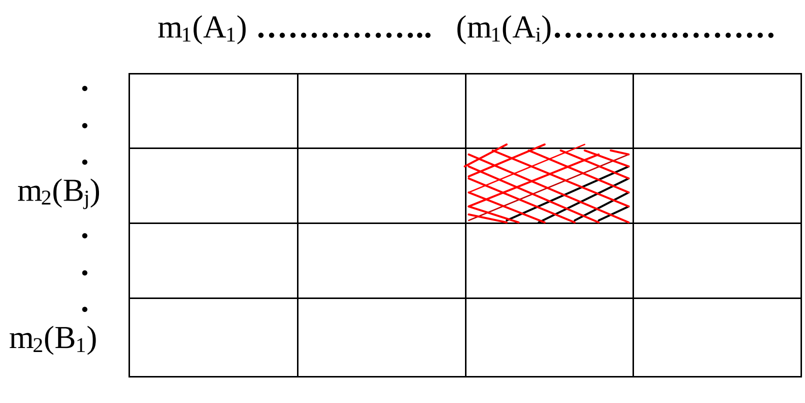

Figure1: Composition of probability mass from sources Bel_1 and Bel_2

The unit square in figure1 represents the total probability mass assigned by both m_1 and m_2 for all of their common subsets. A particular subrectangle within the square, shown as the intersection of the sets A_i and B_j, has committed to it the measure $m_1(A_i)m_2(B_j)$. Likewise, any subset C of U may have one, or more than one, of these rectangles committed to it. Therefore, the total probability mass committed to C will be

$$\sum_{A_i \cap B_j = C} m_1(A_i)m_2(B_j) \qquad \text{..........(1)}$$

Where the summation is over all i and j.

The sum of equation (1) must be normalized to account for the fact that some intersections $A_i \cap B_j = Ø$ will have positive probability which must be discarded. The final form of Dempstr's rule of combination is then given by

$$m_1 \text{ o } m_2 = \frac{\sum_{A_i \cap B_j} m_1(A_i)m_2(B_j)}{\sum_{A_i \cap B_j \neq 0} m_1(A_i)m_2(B_j)} \qquad \text{......... (2)}$$

Where the summations are taken over all i and j.

III. Example

The terrorist group or groups responsible for a certain attack in some country. Suppose any of four known terrorist organizations A,B,C and D could have been responsible for the attack. The possible subsets of U in this case form a lattice of sixteen subsets in figure2.

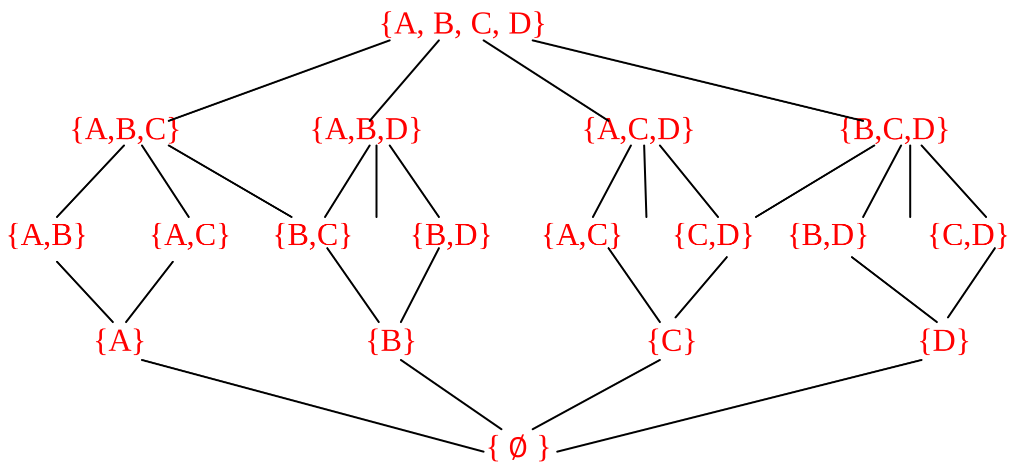

Figure2: Lattice of subsets of the universe U.

Let one piece of evidence supports the belief that groups A and C were responsible to a degree of of $m_1(\{A,C\})$ =0.6, and another source of evidence disproves the belief that C was involved and therefore supports the belief that the three organizations A,B and D were responsible; that is m2({A,B,D}) = 0.7. To obtain the pooled evidence, we compute the following quantities and summarized in Table.

$m_1 \text{ o } m_2(\{A\}) = (0.6)^* (0.7) = 0.42$

$m_1 \text{ o } m_2(\{A,C\}) = (0.6)^*(0.3) = 0.18$

$m_1 \text{ o } m_2(\{A,B,D\}) = (0.4)^*(0.7) = 0.28$

$m_1 \text{ o } m_2(\{U\}) = (0.4)^*(0.3) = 0.12$

$m_1 \text{ o } m_2 = 0$ for all other subsets of U

$Bel_1(\{A,C\}) = m(\{A,C\}) + m(\{A\}) + m(\{C\})$

TABLE: Tableau of combined values of belief for m_1 and m_2

		m_2	
		{A,B,D}(0.7)	U(0.3)
m_1	{A,C} (0.6)	{A}(0.42)	{A,C}(0.18)
	U(0.4)	{A,B,D}(0.28)	U(0.12)

IV. Conclusion

Since much of the knowledge we deal with is uncertain in nature, a number of our beliefs must be tenuous. Our conclusions are often based on available evidence and experience, which is often far from complete. The conclusions are, no more than educated guesses. In a great many situations it is possible to obtain only partial knowledge concerning the possible outcome of some event. But, given that knowledge, one's ability to predict the outcome is certainly better than with no knowledge at all. We manage quite well in drawing plausible conclusions from incomplete knowledge and experiences.

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