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NOVEL APPROACHES TO MULTI-CRITERIA DECISION MAKING WITH INCOMPLETE INFORMATION SYSTEM

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Abstract: Our main work in this study is to make a detailed discussion on the multi-criteria decision making with incomplete information systems. At first, an algorithm is constructed to retrieve the missing criteria values by taking into account the local similarity as well as global similarity of each two alternatives. Then, in view of different evaluation information representation, we establish different making methods for the corresponding completed information system. By transforming interval-valued information into intuitionistic fuzzy number, the cosine similarity measure based method is introduced to the decision making problem with interval-valued evaluation information. Moreover, the aggregation operator based method is established for set-valued information. Especially, we propose a novel decision making approach for the hybrid evaluation information from viewpoint of rough set theory. The validity of these decision making methods are demonstrated by corresponding synthetic examples.

Keywords: aggregation operators; cosine similarity measure; incomplete information system; multi-criteria decision making; rough sets

I. INTRODUCTION

Because of the diversity of the practical problem, how to make scientific behaviors has become a cardinal task for practitioners. Scholars have never given up the pursuit of making an ideal decision from theoretic aspects. However, as Herbert Simon pointed out, "most people are only partly rational, and are in fact emotional or irrational in the remaining part of their actions". Ideal solution is extremely intuitive when considering single criterion problems, since we only need to choose the alternative with the highest preference rating. To make a balance among criteria when there are more than two criteria been taking into account, the trade-off approaches usually are favored by many practitioners. This leads to the emergency of multi-criteria decision-making. In view of its potential advantages, this trade-off method has been combined with many theories, as fuzzy set and intuitionistic fuzzy set [1, 2], gray theory [3], entropy theory[4], rough sets[5,6] et al. In addition, multi-criteria decision-making has found its application areas in layout [7, 8], management [9, 10], and so on.

For a practical decision making problem, the number of alternatives is usually finite, the same to that of criteria. Therefore, the evaluation information of multi-criteria decision-making can be expressed by a matrix. In practical decision making process, the acquisition of information always shows some uncertainty. Hence, many scholars discussed the multi-criteria decision making problem from various viewpoints. Such as Chen and Yang [11], Xu and Xia [12] discussed the multi-criteria decision making problems with intuitionistic fuzzy information (See [13] for theory of intuitionistic fuzzy sets). Ye [14], Park et al. [15] and Chen et al. [16] made a careful discussion on interval-valued intuitionistic fuzzy information for multi-criteria decision-making (See [17] for interval-valued intuitionistic fuzzy set theory). He et al. [18] and Wei et al. [19] studied multi-criteria decision making with triangular fuzzy number evaluation information. Chen and Li [20] extended the evaluation information from triangular fuzzy number to triangular intuitionistic fuzzy number for multi-criteria decision-making. Ye [4] investigated it from viewpoint of trapezoidal fuzzy number.

On the other hand, the approach to deal with multi-criteria decision-making is also an interesting thing for many practitioners. As is known to everyone, the approach "Technique for Order Preferences by Similarity to an Ideal Solution" (TOPSIS, for short) proposed by Hwang and Yoon [21], has won its successful applications [15, 22, 23]. Another important trade-off method used widely by many decision makers is aggregation operators. The essence of it is to calculate the whole performance score of each object. Because there is no need to determine the ideal solution, it has been widely applied to multi-criteria decision making problems under various environments [19, 20, 24, 25]. To select the most desirable alternative, other techniques, such as graphic method [26, 27], mathematical programming [28], entropy theory [29], gray relation [30], etc., have encountered their ideal place.

Obviously, all aforementioned issue is established on the hypothesis that the data information is known. Though this can be guaranteed by modern means of storing data, data losing is unavoidable. Once some data information is missing, the decision result would be more uncertain. Hence, in this study our main work is to make a detailed discussion on the multi-criteria decision making with incomplete information system. To recover the missing criteria values, in following section we at first need to calculate the local similarity of each two alternatives. Since the evaluation information is interval-valued or set-valued, the methods of certain similarity calculation are constructed. After that, the global similarity of each two alternatives can be determined with the pre-assumption that the weights of criteria are completely known. On these bases, one can get the similar class of any alternative with missing criteria values. From now on, the missing criteria values can be retrieved efficiently. For the completed information system with interval-valued evaluation information, the extended TOPSIS method is proposed by changing interval-valued values into intuitionistic fuzzy values. By regarding the unified set-valued values as hesitant fuzzy set the aggregation operators is applied to help decision makers select the most desirable alternative. What is more, at the end of this study, we establish a rough set approach to multi-criteria decision making with incomplete hybrid evaluation information where the evaluation information is interval-valued and set-valued.

The remainder of this paper is organized as follows. In Section 2 we make a detailed description of the procedure for retrieving missing criteria values. In Section 3 we focus on the incomplete multi-criteria decision making problems with single representation, i.e., interval-valued and or set-valued evaluation information. In Section 4, for the incomplete multi-criteria decision making problems with hybrid evaluation information, rough set approach is applied to help the decision makers select the most desirable alternative. Finally, we conclude this paper in Section 5.

II. INCOMPLE DATABASE AND ITS COMPLETION

In this section, we make a recall of information system and then propose an approach to discover the missing data for incomplete information system.

Mathematically speaking, an information system is a pair $IS=(U,AT)$, where $U=\{x_1\}_{i=1}^{m}$ is a non-empty finite set of objects called universe, $AT=\{c_i\}_{i=1}^{n}$ is a finite set of criteria, such that $c_j:U\to V_j$ for any $c_j\in AT$, where V_j is called domain of criterion c_j.

If there exist at least one object with respect to some criteria, take x_i and c_j for example, such that $c_j(x_i)$ is unknown, then we call the information system $IS=(U,$ $AT)$ is incomplete, otherwise it is complete [31]. In other words, for the complete information system, all the objects have known criteria values, but there exist some missing values in an incomplete information system.

In what follows we introduce the detailed procedure for retrieving missing criteria values about incomplete information systems. Given that information system $IS=(U,AT)$ is an incomplete information system m objects and n criteria, then the unknown criteria values can be retrieved by following algorithm.

Algorithm 1:

Completion of the incomplete information system

1. Calculate $OSU=\left\{x_i \middle| x_i\in U, \exists c_j\in A \Rightarrow c_j(x_i)=*\right\}$ where "$*$" represents the missing value.
2. Calculate the weights of criteria by some trick, and denoted by $\theta=(\theta_1,\theta_2,...,\theta_n)$ with $\sum\theta_i=1$.
3. Calculate local similarity of x_i and x_j : (1) If $c_p(x_i)$ and $c_p(x_j)$ is rear number, then $s_p(i,j)=1$ for $i=j$ and
$$s_p(i,j)=\begin{cases}1 & if \begin{cases}c_p(x_i)=c_p(x_j)\\ c_p(x_i)\neq *\\ c_p(x_j)\neq *\end{cases}\\ 0.5 & if \quad others\\ 0 & if \begin{cases}c_p(x_i)\neq c_p(x_j)\\ c_p(x_i)\neq *\\ c_p(x_j)\neq *\end{cases}\end{cases}$$
for $i\neq j$. (2) $c_p(x_i)$ is interval-valued or set-valued for any $x_i\in U$, then
$$s_p(x_i)=\begin{cases}1 & if \begin{cases}c_p(x_i)=*\\ c_p(x_j)=*\end{cases}\\ 0.5 & if \quad others\\ \dfrac{|c_p(x_i)\cap c_p(x_j)|}{|c_p(x_i)\cup c_p(x_j)|} & if \begin{cases}c_p(x_i)\neq *\\ c_p(x_j)\neq *\end{cases}\end{cases}$$
4. Calculate global similarity of x_i and x_j by equation
$$S_A(i,j)=\sum_{p=1}^{n}\theta_p s_p(i,j).$$
5. Construct the similarity class of some objects x_i with missing information by $C(x_i)=\left\{x_j \middle| (x_i,x_j)\in T\right\}$, where $T=\left\{(x_i,x_j) \middle| S_A(i,j)..\xi\right\}$ and ξ is the threshold value.
6. For some $c_p(x_i)=*$, at first we compute

$$C_p(x_i)=\{c_p(x_{it})|\ x_{it}\in C(x_i)\wedge c_p(x_{it})\neq *\}.$$

Then replace $c_p(x_i)$ by $[\min C_p(x_i),\ \max C_p(x_i)]$ if $c_p(x_i)$ is a rear number and $a_p(x_i)=\bigcup_t c_p(x_{it})$ if $c_p(x_i)$ is interval-valued or set-valued.

In what follows, in order to demonstrate the above proposed unknown information retrieval method, we consider an incomplete information system with five objects and seven criteria.

Example 2.1 Table 1 is an incomplete information system, where the members of V_3 and V_4 is interval-valued and the members of V_5 and V_6 is set-valued.

Table 1: an incomplete information system

	c_1	c_2	c_3	c_4	c_5	c_6
x_1	3	2	*	*	1,3	3
x_2	2	*	[0, 1.5]	[0.5, 1]	1,2	*
x_3	1	2	*	[1.5, 2]	1,2,3	*
x_4	*	0	[1, 2]	*	1,2	2,3
x_5	2	1	[0.5,1.5]	[0.3,1.8]	3	1,2

Obviously, we have that $OSU=\{x_1,x_2,x_3,x_4\}$. What is more, suppose that the weight of criterion a_p is $\theta_p=1/6$ for $p=1,2,\cdots,6$. By computing we have that for criterion c_1,

$$s_1=\begin{pmatrix} 1 & 0 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0.5 & 1 \\ 0 & 0 & 1 & 0.5 & 0 \\ 0.5 & 0.5 & 0.5 & 1 & 0.5 \\ 0 & 1 & 0 & 0.5 & 1 \end{pmatrix}$$

Analogous, we can obtain $s_2,\ s_3,\cdots,s_6$. What is more, the global similarity is

$$S_A=\begin{pmatrix} 1 & 0.3889 & 0.6111 & 0.3056 & 0.6667 \\ 0.3889 & 1 & 0.4444 & 0.5417 & 0.5000 \\ 0.6111 & 0.4444 & 1 & 0.4444 & 0.1944 \\ 0.3056 & 0.5417 & 0.4444 & 1 & 0.2778 \\ 0.6667 & 0.5000 & 0.1944 & 0.2778 & 1 \end{pmatrix}$$

If we take $\xi=0.4$, then

$$C(x_1)=\{x_1,x_3,x_5\} \qquad C(x_2)=\{x_2,x_3,x_4,x_5\}$$
$$C(x_3)=\{x_1,x_2,x_3,x_4\} \qquad C(x_4)=\{x_2,x_3,x_4\}$$

Therefore, the completion of the incomplete information system showed in Table 1 can be expressed as follows.

Table 2: the completion of incomplete information system

	c_1	c_2	c_3	c_4	c_5	c_6
x_1	3	2	[0.5,1.5]	[0.3,2]	1,3	3
x_2	2	[0,2]	[0, 1.5]	[0.5,1]	1,2	1,2,3
x_3	1	2	[0,2]	[1.5,2]	1,2,3	2,3
x_4	[1,2]	0	[1,2]	[0.5,2]	1,2	2,3
x_5	2	1	[0.5,1.5]	[0.3,1.8]	3	1,2

III. DECISION APPROACHES TO INCOMPLE INFORMATION SYSTEMS WITH SINGLE EVALUATION INFORMATION

Because of the uncertainty of knowledge acquisition, the evaluation information of alternatives under different criterion usually have different expression format. For example, the evaluation information under one criterion may be interval-valued, while it is set-valued for another criterion. Here we make a detailed discussion on the multi-criteria decision making problems with single evaluation information representation.

A. Interval-valued evaluation information

In this subsection, we pay our attention to the interval-valued evaluation information for multi-criteria decision making problem. First, we construct the detailed procedure for multi-criteria decision making with incomplete data where the evaluation information is interval-valued.

Given that for an incomplete information system with m objects and n criteria, the evaluation information e_{ij} for $i=1,2,\ \cdots,m$ and $j=1,2,\cdots,n$ is an interval-valued. That is to say, $e_{ij}=[e_{ij}^l,e_{ij}^r]$ with condition $e_{ij}^r\geq e_{ij}^l\geq 0$. Here, e_{ij} is equal to $c_j(x_i)$. Then we construct the detailed procedure for decision making as follows.

Algorithm 2:

Approach to multi-criteria decision making with incomplete interval data

1. Complete the incomplete information system by Algorithm 1.
2. In general, $e_{ij}=[e_{ij}^l,e_{ij}^r]$ is not necessary located in unity interval [0, 1]. To make a comparing with the ideal solution described follows by means of weighted cosine similarity measure, in this study we replace e_{ij} by $\tilde{e}_{ij}=[\tilde{e}_{ij}^l,\tilde{e}_{ij}^r]$, where
$$\tilde{e}_{ij}^l=\frac{e_{ij}^l}{\max_i e_{ij}^r} \text{ and } \tilde{e}_{ij}^r=\frac{e_{ij}^r}{\max_i e_{ij}^r}.$$
3. Let $\tilde{e}_{ij}=(\tilde{e}_{ij}^l,1-\tilde{e}_{ij}^r)$ for $i=1,2,\cdots,m$, $j=1,2,\cdots,n$. Notice that Bustince and Burillo [32] have proved that intuitionistic fuzzy sets, to some degree, are interval-

valued fuzzy sets. So, the ideal solution is expressed by intuitionistic fuzzy sets.

4. Determine the ideal solution:

$$ide^* = (ide_1^*, ide_2^*, \cdots, ide_n^*)$$

where ide_j^* is the ideal point under c_j for $j = 1, 2, \cdots, n$ .If c_j is benefit, then $ide_j^* = (1, 0)$,and $ide_j^* = (0, 1)$ if c_j is cost.

5. Compute the distance between x_i and ideal solution ide^* by equation $C_s(x_i, ide^*) = \sum_{j=1}^{n} \theta_j \Delta_j$, where

$$\Delta_j = \frac{\tilde{e}_{ij}^l}{\sqrt{(\tilde{e}_{ij}^l)^2 + (\tilde{e}_{ij}^r)^2 + (1 - \tilde{e}_{ij}^r - \tilde{e}_{ij}^l)^2}}$$

if c_j is a benefit criterion and

$$\Delta_j = \frac{\tilde{e}_{ij}^r}{\sqrt{(\tilde{e}_{ij}^l)^2 + (\tilde{e}_{ij}^r)^2 + (1 - \tilde{e}_{ij}^r - \tilde{e}_{ij}^l)^2}}$$

if c_j is a cost criterion.

6. Rank $C_s(x_i, ide^*)$ for $i = 1, 2, \cdots, m$.

With aforementioned steps, the decision makers can select the most desirable alternative successfully.

B. Set-valued evaluation information

What follows is the procedure for multi-criteria decision making with incomplete data where the evaluation information is set-valued. For an incomplete information system with m alternatives and n criteria, let e_{ij} for $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$ be set-valued, i.e., $e_{ij} = \{e_{ij}^1, e_{ij}^2, \cdots, e_{ij}^{\delta_{ij}}\}$, where δ_{ij} is the number of possible members of e_{ij} . Following is the detailed procedure.

Algorithm 3:

Approach to multi-criteria decision making with incomplete set-valued data

1. Complete the incomplete information system by Algorithm 1.
2. Here we suppose that e_{ij}^* for $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$ is real number, then replace e_{ij}^* by $\hat{e}_{ij}^*$, where $\hat{e}_{ij}^* = \dfrac{e_{ij}^*}{\max\limits_j e_{ij}}$.
3. Aggregating evaluation information of x_i under all criteria by equation

$$E(x_i) = \bigcup_{j=1}^{n} \left\{ 1 - \prod_{j=1}^{n} (1 - \gamma_j)^{\theta_j} \mid \gamma_j \in \hat{e}_{ij} \right\}$$

Notice that the normalized domain $V_j = \{\hat{e}_{1j}, \hat{e}_{2j}, \cdots, \hat{e}_{mj}\}$ can be regarded as the collection of fuzzy sets $\hat{e}_{ij}$ for $i = 1, 2, \cdots, m$, in which case $\hat{e}_{ij}$ is equal to the hesitation fuzzy set proposed by Torra [33].

4. Compute $\tilde{E}(x_i)$, the performance score of alternative x_i for $i = 1, 2, \cdots, m$, by equation

$$\tilde{E}(x_i) = \frac{1}{|E(x_i)|} \sum_{e_i^* \in E(x_i)} e_i^* .$$

Obviously, above equation constitutes the hesitant fuzzy weighted averaging operator defined by Xia and Xu [34].

5. Rank $\tilde{E}(x_i)$ for $i = 1, 2, \cdots, m$.

With aforementioned steps, the decision makers can select the most desirable alternative successfully.

IV. ROUGH SET APPROACH TO INCOMPLETE INFORMATION SYSTEM WITH HYBRID INFORMATION

In view of the process of multi-criteria decision making, the essence of which is to establish a technique for decision makers to select the most desirable alternative. After a series of calculation, we can assign a real number to each possible alternative, the so- called performance score. In fact, the procedure for getting the performance score is to comparing with each alternative under all criteria by means of trade-off technique. If the criterion were benefit, then the larger the value of corresponding alternative with respect to this criteria, the better the alternative would be. Analogous, if the criterion is a cost criterion, then the smaller the better. Under such circumstance, the selection of the most desirable alternative is to construct the partial ordering relationship among alternative under all criteria.

Generally, the theory of rough sets [35-37] is based on partition mechanism where the binary relation plays a vital role. The great advantage of which is that it does not need any prior knowledge. Hence, it has been applied widely and successfully in many areas. By taking the limitation of equivalence binary relation into consideration, Greco et al. proposed dominance relation based rough set model [5, 38, 39].

Next we introduce a decision making approach for multi-criteria decision making with incomplete hybrid information from viewpoint of rough set. Before detailed description, let A_b is the set of benefit criteria and A_c is the set of cost criteria. Then we construct the dominance relation on criteria set AT of the incomplete information system as follows.

Definition 4.1 Let $IS=(U,AT)$ be an information system with m alternatives and n criteria, then the dominance relation on criteria set A_b can be defined as

$$R_{A_b}^{\pm}=\left\{(x_i,x_j)\left|\left\{c_p(x_i)\ldots_p c_p(x_j)\wedge c_p\in A_b\right\}\right.\right\}$$

and the dominance relation on criteria set A_c can be defined as

$$R_{A_c}^{\pm}=\left\{(x_i,x_j)\left|c_q(x_j)\ldots_q c_q(x_i)\wedge c_q\in A_c\right.\right\}$$

Where "$\ldots_p$" is the partial order relation on V_p, and the same to that of "$\ldots_q$".

Mathematically speaking, if the members of V_p are interval-valued,

$$c_p(x_i)\ldots_p c_p(x_j)\Leftrightarrow\begin{cases}c_p^l(x_i)\ldots c_p^l(x_j)\\ c_p^r(x_i)\ldots c_p^r(x_j)\end{cases},$$

where $c_p(x_i)=[c_p^l(x_i),c_p^r(x_i)]$, $c_p(x_j)=[c_p^l(x_j),c_p^r(x_j)]$.

If the members of V_p are set-valued, then $c_p(x_i)\ldots_p c_p(x_j)$ if and only if $c_p(x_i)\supseteq c_p(x_j)$.

Certainly, if a multi-criteria decision making system contains not only benefit criteria, but also cost criteria, then the dominance relation on it can be defined as follows.

Definition 4.2 Let $IS=(U,AT)$ be an information system with $A_b\neq\varnothing$ and $A_c\neq\varnothing$, then the dominance relation on AT can be defined as

$$\begin{aligned}R_{AT}^{\pm}&=R_{A_b}^{\pm}\cap R_{A_c}^{\pm}\\&=\left\{(x_i,x_j)\left|\begin{cases}c_p\in A_b\Rightarrow c_p(x_i)\ldots_p c_p(x_j)\\ c_p\in A_c\Rightarrow c_p(x_j)\ldots_p c_p(x_i)\end{cases}\right.\right\}\end{aligned}$$

On this basis, the dominance class of each alternative can be expressed as

$$[x_i]_{AT}^{\pm}=\left\{x_j\left|(x_j,x_i)\in R_{A_b}^{\pm}\wedge(x_i,x_j)\in R_{A_c}^{\pm}\right.\right\}.$$

From now on, we propose the dominance degree of one object to another object constructed by Xu [40] as

$$D_g(x_i,x_j)=1-\frac{\left|[x_i]_{AT}^{\pm}\cap(\sim[x_j]_{AT}^{\pm})\right|}{|U|},$$

where "$\sim$" is the complementary operation, i.e., $\sim[x_j]_{AT}^{\pm}=\{x_i|\ x_i\notin[x_j]_{AT}^{\pm}\}$. So much for this, the global dominance degree of x_i for $i=1,2,\cdots,m$ is calculated by equation

$$D_{AT}(x_i)=\frac{1}{|U|}\sum_{x_j\in U}D_g(x_i,x_j).$$

In view of its meaning for objects ranking, The dominance degree "D_g" as well as global dominance degree "D_A" is based on hypothesis that the criteria values are all known and all criteria are benefit criteria. Here, we take two situations into account, there exist unknown criteria values, and the criteria set contains cost and benefit criteria. Suppose that for an incomplete information system, $AT=\{c_1,c_2,\cdots,c_n\}$ is the set of criteria, such that $AT=A_b\cup A_c$ and $A_b\cap A_c=\varnothing$. $V_p=\{e_{ip}|\ i=1,2,\cdots,m\}$ is the domain of criterion c_p such that e_{ip} takes the representation of real number, interval-valued, set-valued et al. for $p=1,2,\cdots,n$. Therefore, a new method of multi-criteria decision making with incomplete hybrid evaluation information can be constructed as follows.

Algorithm 4: Approach to multi-criteria decision making with incomplete hybrid evaluation information

1. Complete the incomplete information system by Algorithm 1.
2. Determine the benefit criteria A_b and cost criteria A_c according to the practical problems, such that $A_b\cup A_c=A$ and $A_b\cap A_c=\varnothing$.
3. Determine the dominance relation $R_{AT}^{\pm}$ by Definition 4.2.
4. Calculate the dominance class of each $x_i\in U$.
5. Calculate $D_g(x_i,x_j)$ for $i,j=1,2,\cdots,m$.
6. Calculate $D_{AT}(x_i)$ for $i=1,2,\cdots,m$.
7. Rank $D_{AT}(x_i)$ for $i=1,2,\cdots,m$.

Example 4.1 Table 3 is the completion of an incomplete information system with five alternatives and four criteria. Here we suppose that benefit criteria is $A_b=\{c_1,c_2,c_3\}$ and cost criteria is $A_c=\{c_4\}$.

Table 3: an information system

	c_1	c_2	c_3	c_4
$\mathbf{x_1}$	2	0.7	[0.4, 0.7]	1
$\mathbf{x_2}$	3	0.8	[0.6, 0.8]	1, 2
$\mathbf{x_3}$	2	0.6	[0.1, 0.6]	1
$\mathbf{x_4}$	2	0.7	[0.8, 0.9]	1
$\mathbf{x_5}$	1	0.6	[0.1, 0.6]	1,2

Next, we make an analysis for this decision information system. By computing, we have that

$$R_{AT}^{\pm}=\begin{Bmatrix}(x_1,x_1) & (x_1,x_3) & (x_1,x_5) & (x_2,x_2)\\ (x_2,x_5) & (x_3,x_3) & (x_3,x_5) & (x_4,x_1)\\ (x_4,x_3) & (x_4,x_4) & (x_4,x_5) & (x_5,x_5)\end{Bmatrix}$$

Therefore,

$[x_1]^{\pm}_{AT} = \{x_1, x_4\}$ $\quad [x_2]^{\pm}_{AT} = \{x_2\}$

$[x_3]^{\pm}_{AT} = \{x_1, x_3, x_4\}$ $\quad [x_4]^{\pm}_{AT} = \{x_4\}$

$[x_5]^{\pm}_{AT} = \{x_1, x_2, x_3, x_4, x_5\}$

With foregoing definitions, we have that

$$D_g = \begin{pmatrix} 1 & 0.6 & 1 & 0.8 & 1 \\ 0.8 & 1 & 0.8 & 0.8 & 1 \\ 0.8 & 0.4 & 1 & 0.6 & 1 \\ 1 & 0.8 & 1 & 1 & 1 \\ 0.6 & 0.6 & 0.4 & 0.8 & 1 \end{pmatrix}$$

Hence,

$$D_{AT}(x_1) = 0.88 \quad D_{AT}(x_2) = 0.88$$
$$D_{AT}(x_3) = 0.76 \quad D_{AT}(x_4) = 0.96$$
$$D_{AT}(x_5) = 0.72$$

From above we have that $x_4 \succ x_1 = x_2 \succ x_3 \succ x_5$, in which case the alternative x_4 is the most desirable alternative.

Obviously, the rough set approach to multi-criteria decision making reduces subjective factors during decision process, such as the determination of criteria weights, etc. Attribute reduction [37, 41] is one of the important knowledge of rough set theory. Its basic idea is to delete the redundant attributes without changing the classification ability. Therefore, for the problem of multi-criteria decision making, one can reduce redundant criteria before computing the dominance class for each alternative. The reason of it is that the dominance class of each alternative directly influences the possible alternative ranking results. Hence, the Algorithm 4 can be changed into following steps:

Algorithm 4*: changing version of Algorithm 4.

1. Complete the incomplete information system by Algorithm 1.
2. Determine the benefit criteria A_b and cost criteria A_c according to the practical problems, such that $A_b \cup A_c = A$ and $A_b \cap A_c = \varnothing$.
3. Determine the dominance relation on criteria set A_b by Definition 4.1.
4. Look for the reduction of the completed incomplete information system, and denoted by $A^{(1)}, A^{(2)}, \cdots, A^{(k)}$, where k is the number of possible reductions.
5. For any $A^{(*)}$, please compute $R^{\pm}_{A^{(*)}}$ and $[x_i]^{\pm}_{A^{(*)}}$ for $i = 1, 2, \cdots, m$ by Definition 4.2.
6. Calculate $D_g(x_i, x_j)$ for $i, j = 1, 2, \cdots, m.$
7. Calculate $D_{A^{(*)}}(x_i)$ for $i = 1, 2, \cdots, m.$
8. Rank $D_{A^{(*)}}(x_i)$ for $i = 1, 2, \cdots, m$.

Example 4.2(Continued from Example 4.1) by computing one get that $[x_i]^{\pm}_{A^{(*)}} = [x_i]^{\pm}_{AT}$ for $i = 1, 2, \cdots, 5$, where $A^{(*)} = \{c_1, c_3, c_4\}$. Hence, for the multi-criteria decision making problem, the criteria c_1, c_3 and c_4 is enough for decision makers to select the most desirable alternative.

V. CONCLUSIONS

In this paper, we discussed the approaches to multi-criteria decision making with incomplete evaluation information. To retrieve the missing criteria values, we first introduced the global similarity between two alternatives by considering the weighted local similarity of them. Then, the pre-established threshold can be applied to determine the similar class of each alternative. Once the incomplete information system was completed, different approaches for multi-criteria decision making with different evaluation information were proposed, such as cosine similarity measure based method for interval-valued information system and aggregation operator based method for set-valued information system. Especially, rough set approach was established for the multi-criteria decision making problems with hybrid evaluation information. It should be pointed that the validity of all proposed approached are examined by corresponding examples.

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