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Multichannel Blind Deconvolution of the Arterial Pressure using Ito Calculus Method

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Abstract: Multichannel Blind Deconvolution (MBD) is a powerful tool particularly for the identification and estimation of dynamical systems in which a sensor, for measuring the input, is difficult to place. This paper presents Ito calculus method for the estimation of the unknown time-varying coefficient. The arterial network is modelled as a Finite Impulse Response (FIR) filter with unknown coefficients. A new tool for estimation of both the central arterial pressure and the unknown channel dynamics has been developed. The convolution process is modelled as a Finite Impulse Response (FIR) filter with unknown coefficients. The source signal is also unknown. Assuming that one of the FIR filter coefficients are time varying, we have been able to get accurate estimation results for the source signal, even though the filter order is unknown. The time varying filter coefficients have been estimated through the SC algorithm, and we have been able to deconvolve the measurements and obtain both the source signal and the convolution path. The positive results demonstrate that the SC approach is superior to conventional methods.

Keywords: Finite Impulse Response (FIR), Multichannel Blind Deconvolution (MBD), Stochastic Calculus (SC).

1- Introduction

The framework for this methodology is based on a multi-channel blind deconvolution (MBD) technique that has been reformulated to use Stochastic Calculus (SC). The technique is based on (MBD) of dynamic system, in which, as shown in Figure (1), two measured outputs (peripheral artery pressure PAP waveforms from the femoral AP waveform and radial AP waveform) of a single input (central AP) are mathematically analyzed, in order to reconstruct the common unobserved input within an arbitrary scale factor.

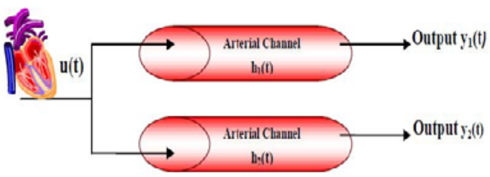


Figure (1): The two measured and sampled peripheral AP waveforms $y_1(t)$ and $y_2(t)$ are modeled as outputs of two unknown arterial channels $h_1(t)$ and $h_1(t)$ driven by common input u(t).

In this paper, we suggest characterizing the arterial channels of the single-input, multi-output system model of the arterial tree by linear and time-variant FIR filters. There are many methods for solving problems of estimation time varying coefficients. But we introduce method that is based on stochastic calculus [1, 2]. If we make only one of the FIR filter parameters changing over time, then the problem is handled by the Ito calculus [2, 3].

We introduce the stochastic calculus based methods [12] that are used to estimate the time-varying parameters/coefficients. This way, the ambiguity in the order of the impulse response is compensated by the time variations of the filter parameters [4, 5, 6, 7].

The method is Ito-calculus based approach [13, 14, 15]. Assuming a slow time-varying regression coefficient, we assume that it is evolving according to the Ornstein-Uhlenbeck (OU) process. The unknown parameters of the OU process are estimated by the maximum likelihood method.

The proposed method is then applied to noninvasive monitoring of the cardiovascular system of the swine. The arterial network is modeled as a multichannel system where the aortic AP is the input and pressure profiles measured at different branches of the artery, e.g., radial and femoral arteries, are the outputs. The proposed methods would allow us to estimate both the waveform of the input pressure and the arterial channel dynamics from outputs obtained with noninvasive sensors placed at different branches of the arterial network. Numerical examples verify the major theoretical results and the feasibility of the method.

2- System Identification and Methodology

The cardiovascular system is topologically analogous to a multichannel dynamic system. Pressure wave emanating from a common source, the heart, is broadcast and transmitted through the many vascular pathways. Therefore, noninvasive circulatory measurements taken at different locations as shown in Figure (3) can be treated as multichannel data and processed with an MBD algorithm.

Our technique applies a novel MBD method to two peripheral AP waveforms (outputs) in order to reconstruct the central AP waveform (input) within an arbitrary scale factor. The channels relating the common input to each output represent the vascular dynamic properties of different arterial tree paths as shown Figure (2) and are assumed to be characterized by finite impulse responses (FIRs). The filters contain many parameters. We estimate the coefficients by the conventional method in Section (2.1), and then we assume that one of the coefficients is varying with time. This way we will be able to compensate for the small number of FIR filter parameters and for the time variation of the channel.

There are many methods for solving problems of estimation time-varying coefficients. But we introduce a method that is based on stochastic calculus in Section (2.2). Assuming a slow time-varying regression coefficient, we assume that it is evolving according to the Ornstein-Uhlenbeck (OU) process. The unknown parameters of the OU process are estimated by the maximum likelihood method. Finally, through the inversion of the FIR filter, we get the original source signal (Central/Aortic AP) within an arbitrary scale factor.

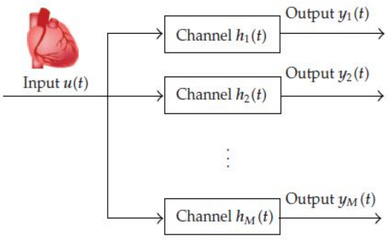


Figure (2): The M (>1) measured and sampled peripheral AP waveforms [$y_1(t)$, 1 < i < M] are modeled as outputs of M unknown channels driven by the common input [u(t)].

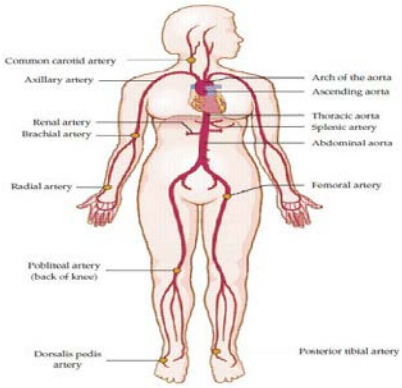


Figure (3): The aorta and arteries. Solid gold dots indicate pulse points in arteries. These are areas in which the pulse (expansion and contraction of a superficial artery) can be felt.

We will be working in the probability space $\Omega, \mathcal{F}, \mathcal{P}$. To simplify the exposure, we will assume that we have only two measurements outputs of a modulated version of the source signal that are given as follows:

$$y_1(t) = h_1(t) * u(t) + \varepsilon_1(t),$$
$$y_2(t) = h_2(t) * u(t) + \varepsilon_2(t), \quad (2.1)$$

Where $u(t)$ is the unknown source signal (central AP), $h_1(t)$ and $h_2(t)$ are unknown filters (hemodynamic response at time *t*) or modulating paths, "$*$" is the convolution operation, $y_1(t)$ (femoral AP) and $y_2(t)$ (radial AP) are the observed measurements, $\varepsilon_1(t)$ and $\varepsilon_1(t)$ are the measurements noise. The objective is to deconvolve and to estimate $y_1(t)$ and $y_2(t)$ to estimate $u(t)$. If we convolve $y_1(t)$ with $h_2(t)$, we will get:

$$h_2(t) * y_1(t) = h_2(t) * (h_1(t) * u(t)) + h_2(t) * \varepsilon_1(t) \quad (2.2)$$

Since the convolution is a commutative operation, then exchanging $h_1(t)$ and $h_2(t)$ and on the right hand side, we get

$$h_2(t) * y_1(t) = h_1(t) * (h_2(t) * u(t)) + h_2(t) * \varepsilon_1(t)$$
$$= h_1(t) * y_2(t) - h_1(t) * \varepsilon_2(t) + h_2(t) * \varepsilon_1(t) \quad (2.3)$$

Thus,

$$h_2(t) * y_1(t) = h_1(t) * y_2(t) - h_1(t) * \varepsilon_2(t) + h_2(t) * \varepsilon_1(t) \quad (2.4)$$

Note that this equation does not include the input *u(t)*. It represents the constraints among the channel dynamics or filters and observed output. Substituting a measured time series of output data for $y_1(t)$ and $y_2(t)$, the above equation can be solved for the unknown parameters involved in $h_1(t)$ and $h_2(t)$. Once the filters are obtained, we will use their inverses to find an estimate for the source signal. To simplify the exposure further, assume that the modulating filters, that represent the signal paths or channel dynamics, are second-order linear time invariant and have the Z transforms as follows:

$$h_1(z) = 1 + \beta_1 z^{-1} + \beta_2 z^{-2} \quad (2.5)$$

That is,

$$y_1(k) = u(k) + \beta_1 u(k-1) + \beta_2 u(k-2) + \varepsilon_1(k) \tag{2.6}$$

And in matrix format for N data points:

$$\begin{bmatrix} y_1(2) \\ y_1(3) \\ \cdots \\ y_1(N-1) \end{bmatrix} = \begin{bmatrix} \beta_2 & \beta_1 & 1 & 0 & \cdots & 0 \\ 0 & \beta_2 & \beta_1 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \beta_2 & \beta_1 & 1 \end{bmatrix} \begin{bmatrix} u(0) \\ u(0) \\ \cdots \\ u(N-1) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(2) \\ \varepsilon_1(3) \\ \cdots \\ \varepsilon_1(N-1) \end{bmatrix} \tag{2.7}$$

That is,

$$\underline{Y_1} = H_1\, \underline{U} + \underline{\varepsilon_1} \tag{2.8}$$

Where,

$$\underline{Y_1} = [y_1(2)\ y_1(3)\ \ldots\ y_1(N-1)]^T$$
$$\underline{U} = [u(0)\ \ u(1)\ \ \ldots\ \ u(N-1)]^T \tag{2.9}$$
$$\underline{\varepsilon_1} = [\varepsilon_1(2)\ \varepsilon_1(3)\ \ldots\ \varepsilon_1(N-1)]^T$$

$$H_1 = \begin{bmatrix} \beta_2 & \beta_1 & 1 & 0 & \vdots & 0 \\ 0 & \beta_2 & \beta_1 & 1 & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & 0 & \beta_2 & \beta_1 & 1 \end{bmatrix} \tag{2.10}$$

And "T" stands for transpose.

Similarly,

$$h_2(z) = 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} \tag{2.11}$$

That is,

$$y_2(k) = u(k) + \alpha_1 u(k-1) + \alpha_2 u(k-2) + \varepsilon_2(k) \tag{2.12}$$

Where $h_1(z)$ and $h_2(z)$ are the Z transforms of the discrete versions of $h_1(t)$ and $h_2(t)$ respectively. Since,

$$h_2(t) * y_1(t) = h_1(t) * y_2(t) - h_1(t) * \varepsilon_2(t) + h_2(t) * \varepsilon_1(t) \tag{2.13}$$

Taking the Z transform of the discrete version of both sides we get:

$$h_2(z) y_1(z) = h_1(z) y_2(z) - h_1(z)\varepsilon_2(z) + h_2(z)\varepsilon_1(z) \tag{2.14}$$

In the time domain, we get the equation:

$$y_1(k) + \alpha_1 y_1(k-1) + \alpha_2 y_1(k-2) = y_2(k) + \beta_1 y_2(k-1) + \beta_2 y_2(k-2) + \begin{bmatrix} -\varepsilon_2(k) \ -\beta_1\varepsilon_2(k-1) \\ -\beta_2\varepsilon_2(k-2) + \varepsilon_1(k) \\ +\alpha_1\varepsilon_1(k-1) \\ +\alpha_2\varepsilon_1(k-2) \end{bmatrix} \quad (2.15)$$

2.1- A Conventional Method for the Estimation of the System

The familiar scalar regression format as shown in (Eq. (2.15)) is the shape of a regression equation with a correlated error (colored noise). Unless we take this into consideration, the ordinary least square (OLS) method will yield biased estimates for the unknown coefficients $\alpha_1, \alpha_2, \beta_1, \beta_2$.

Rearrange the (Eq. (2.15)), we get:

$$[y_1(k) - y_2(k)] = -\alpha_1 y_1(k-1) - \alpha_2 y_1(k-2) + \beta_1 y_2(k-1) + \beta_2 y_2(k-2) + \begin{bmatrix} -\varepsilon_2(k) \ -\beta_1\varepsilon_2(k-1) \\ -\beta_2\varepsilon_2(k-2) + \varepsilon_1(k) \\ +\alpha_1\varepsilon_1(k-1) \\ +\alpha_2\varepsilon_1(k-2) \end{bmatrix} \quad (2.16)$$

For the general case where the order of the FIR filters is I and J we get:

$$[y_1(k) - y_2(k)] = -\sum_{i=1}^{I}\alpha_i y_1(k-i) + \sum_{j=1}^{J}\beta_j y_2(k-j) + \left\{\left[-\sum_{j=1}^{J}\beta_j\varepsilon_2(k-j) + \sum_{i=1}^{I}\alpha_i\varepsilon_1(k-i)\right] + [\varepsilon_1(k) - \varepsilon_2(k)]\right\} \quad (2.17)$$

This could be approximated as:

$$[y_1(k) - y_2(k)] \approx -\sum_{i=1}^{I}\alpha_i y_1(k-i) + \sum_{j=1}^{J}\beta_j y_2(k-j) + \varepsilon(k) \quad (2.18)$$

Since the filter orders are unknown, one could use the corrected Akaike information criterion AIC_c to determine both " I " and " J ". Assume that the error term $\varepsilon(k)$ is zero mean Gaussian with variance σ^2, the AIC_c is defined as [8]:

$$AIC_c = n(ln\sigma^2 + I) + \frac{2n(p+I)}{(n-p-2)} \quad (2.19)$$

Where n is the number of observations, $p = I + J$ is a number of unknown, and σ^2 is an estimate of the error variance. We choose the order p such that AIC_c is minimized.

Once the coefficients of the FIR filter are estimated, we use inverse filtering to find an estimate for the source signal U as follows:

$$\underline{\hat{U}} = \hat{H}_1^T (\hat{H}_1 \hat{H}_1^T)^{-1} \underline{Y_1} \quad (2.20)$$

Where,

$$\underline{Y_1} = [y_1(2)\ y_1(3)\ \ldots\ y_1(N-1)]^T$$
$$\underline{\hat{U}} = [\hat{u}(0)\ \ \hat{u}(1)\ \ldots\ \ \hat{u}(N-1)]^T \quad (2.21)$$

And,

$$\hat{H}_1 = \begin{bmatrix} \hat{\beta}_2 & \hat{\beta}_1 & 1 & 0 & \vdots & 0 \\ 0 & \hat{\beta}_2 & \hat{\beta}_1 & 1 & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & 0 & \hat{\beta}_2 & \hat{\beta}_1 & 1 \end{bmatrix} \quad (2.22)$$

The symbol "∧" on top of the variable refers to estimation. For example $\hat{\beta}_2$ and $\hat{\beta}_1$ are the estimates for β_2 and β_1 respectively. Due to its simplicity, the above mentioned method is the one that is commonly used [9].

2.2- Ito Calculus for the Estimation of the Unknown Time-Varying Coefficient

The linear filter assumption is just an approximation to reality. Sometimes the media is nonlinear, time varying, random, or all. The measured signals are usually noisy. The filter order is usually unknown. All the factors suggest that the FIR filter model is an approximation. To compensate for these assumptions, we suggest to make one or some of the unknown coefficients varying with time, i.e., $(\alpha_1(t), \alpha_2(t), \beta_1(t), \beta_2(t))$ or in discrete form $(\alpha_1(k), \alpha_2(k), \beta_1(k), \beta_2(k))$. We restrict ourselves to only one time-varying parameter. Now the problem becomes that of the estimation of the unknown time-varying coefficients. The details of the estimation procedure are given in this section.
We now recast the problem in the format that could be handled by the Ito calculus. Using (Eq. (2.16)), the observed signal, $p_y(k)$ with (d-1) components, could be modeled as follows:

$$p_y(k) = \sum_{i=1}^{d} n_i(k)\, s_i(k) \quad (2.23)$$

where $n_i(k)$, i>1 , is the i^{th} unknown time-varying coefficient, and the random error has been included in each $n_i(k)$ as follow:

$p_y(k)$=[$y_1(k) - y_2(k)$], $n_2(k)$=-$\alpha_1(k)$, $n_3(k)$=-$\alpha_2(k)$, $n_4(k)$=$\beta_1(k)$, $n_5(k)$=$\beta_2(k)$, $s_2(k)$=$y_1(k-1)$, $s_3(k)$=$y_1(k-2)$, $s_4(k)$= $y_2(k-1)$, and $s_5(k)$= $y_2(k-2)$. (2.24)

This equation will be used if we allow all the coefficients to be time-varying [4].
In the proposed approach, it is assumed that the stochastic processes, $y_1(k-1)$, $y_1(k-2)$, $y_2(k-1)$ and $y_2(k-2)$ are independent. If they are correlated, the correlation coefficients will be known. It is also assumed that a stochastic differential equation (SDE) for each process is known. Usually, but not necessary, an Ornstein-Uhlenbeck (OU) process is assumed to describe the evolution of the processes, $y_1(k-1)$, $y_1(k-2)$, $y_2(k-1)$ and $y_2(k-2)$ [10].
We propose to model the unknown time-varying coefficients as OU processes. The OU models are used when the trend in the time-varying parameter is known or could be guessed. The OU model represents a

signal that is bouncing around its trend [11]. In our case we assume that all the coefficients are constants and only $\hat{\beta}_2$ has *SDE* of the OU form:

$$d\beta_2(t) = c_2(\beta_2(0)\text{-}\beta_2(t))dt + e_2 dW_2(t) \tag{2.25}$$

Where c_2 (drift parameter), e_2 (diffusion parameter) are unknown constants to be estimated, $W_2(t)$ is a Wiener process, and $\beta_2(0)$ is the estimated value through the constant coefficient model of the conventional method.

Using the model of section (2.1), we get

$$y_1(k) - y_2(k) = -\alpha_1 y_1(k-1) - \alpha_2 y_1(k-2) + \beta_1 y_2(k-1) + \beta_2 y_2(k-2) \tag{2.26}$$

Rearrange to separate the measurements of $\beta_2(k)$ we get

$$\beta_2(k) \approx \frac{[y_1(k)-y_2(k)]-[\hat{\alpha}_1 y_1(k-1)-\hat{\alpha}_2 y_1(k-2)+\hat{\beta}_1 y_2(k-1)]}{y_2(k-2)} \tag{2.27}$$

Where $\hat{\alpha}_1, \hat{\alpha}_2$ and $\hat{\beta}_1$ are the estimates of and respectively. These estimators could be obtained through the least square method or any other method.

$\beta_2(k)$ of Eq. (2.27) is a noisy measurement, not real, because the model order is not known and we have approximated the order by 2 for simplicity. If we use $\beta_2(k)$ of Eq. (2.27) we get spikes and erroneous estimates of the input AP. As such an estimate for $\beta_2(k)$ is needed. We do this by modeling the time-varying coefficient $\beta_2(k)$ as an OU process with unknown coefficients. Estimating the coefficients of $\beta_2(k)$ will yield an estimate for $\beta_2(k)$.

Specifically we have discrete measurements of the stochastic process $\beta_2(k)$. We need to estimate the unknown deterministic parameters of this process; mainly c_2 and e_2 of Eq. (2.27). We use the maximum likelihood method to achieve this objective. Other methods could be used [4] as well.

2.2.1- Estimation of the Diffusion Parameter of (2.25)

For an observation period $[0, T]$, squaring both sides of equation (2.25) we get,

$$[d\beta_2(t)]^2 = e_2 dt \tag{2.28}$$

Where we use the properties of the Ito calculus, we get,

$$dt\,dt = 0,\ dt dW_2(t) = 0 \text{ and } dW_2(t)\, dW_2(t) = dt \tag{2.29}$$

Thus,

$$\hat{e_2} = \frac{1}{T}\int_0^1 [d\beta_2(t)]^2\, dt \tag{2.30}$$

2.2.2- Estimation of the Drift Parameter of (2.25)

Following [1, 2], the maximum likelihood estimate of the drift parameter is given as

$$\hat{c_2} = -\frac{\int_0^T \beta_2(t) d\beta_2(t)}{\int_0^T {\beta_2}^2(t) dt - \beta_2(0)\int_0^T \beta_2(t) d(t)} \tag{2.31}$$

Thus, an estimate for $\beta_2(t)$ is obtained by substituting equation (2.30) and equation (2.31) in equation (2.25). Once the parameters are estimated, we use inverse filtering to find U (the aortic pressure waveform) as follow.

$$\underline{\hat{U}} = \hat{H}_1{}^T(\hat{H}_1\hat{H}_1{}^T)^{-1}\,\underline{y_1} \tag{2.32}$$

Where,

$$\underline{y_1} = [y_1(2) \quad y_1(3) \quad \dots \quad y_1(N-1)]^T \tag{2.33}$$

And,

$$\hat{H}_1 = \begin{bmatrix} \hat{\beta}_2(t_1) & \hat{\beta}_1 & 1 & 0 & \vdots & 0 \\ 0 & \hat{\beta}_2(t_2) & \hat{\beta}_1 & 1 & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & 0 & \hat{\beta}_2(t_{N-2}) & \hat{\beta}_1 & 1 \end{bmatrix} \tag{2.34}$$

We note from matrix $\hat{H}_1$ that the parameter $\hat{\beta}_2(t)$ has values that are changing across the sample.

3- Results

To test the proposed approach, first, we took real data from [12] by inserting the graph that contains the measured data on software CURVESCAN and extracting points see Figure (4).

Second, we simulated a set of 300 data points on computer measured data. Multichannel blind deconvolution was experimentally evaluated with respect to measured data in which femoral artery pressure (AP), radial AP waveforms, and aortic pressure waveform were simultaneously measured see Figure (3). Third, we demonstrate the ability of the proposed approach to extract Aorta AP waveform from multichannel. All the calculations in the algorithm were performed under MATLAB (7.2) [16, 17]. Fourth, we evaluated the proposed method by two performance measures.

1- The signal to noise ratio of the estimates (SNRE) was taken as the measure of performance for this evaluation. It is defined as:

$$SNRE = 10\log\frac{\sum_k u^2(k)}{\sum_i[u(i)-\hat{u}(i)]^2} \tag{3.1}$$

Where $\hat{u}(i)$ is the estimated value of the pressure at instant "i", see table (1).

2- The mean absolute percent error (MAPE) was taken as another performance measure for this evaluation, see table (2). It is defined as:

$$MAPE = \left[\frac{1}{N}\sum_{i=1}^{N}\frac{|\hat{u}(i)-u(i)|}{u(i)}\right] \times 10 \tag{3.2}$$

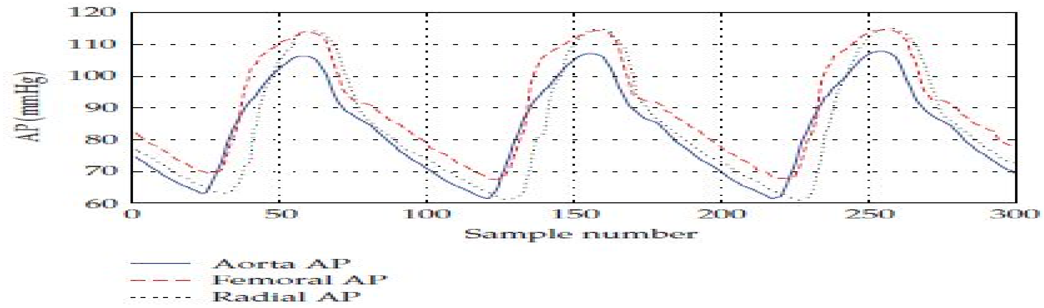


Figure (4): Segments of measured central arterial pressure AP, femoral AP, and radial AP waveforms from one swine dataset.
one swine dataset.

Methods	SNRE $_{0.0}$	SNRE $_{0.012}$	SNRE $_{0.024}$	SNRE$_{0.037}$	SNRE $_{0.049}$	SNRE$_{0.0615}$
FIR-2 order	9.006 db	7.061 db	6.730 db	6.380 db	6.073 db	5.995 db
FIR-4 order	12.349 db	11.248 db	11.076 db	10.974 db	10.582 db	10.108 db
Stochastic Calculus	23.296 db	22.049 db	21.047 db	20.727 db	19.400 db	18.199 db

Table (1): The first performance measure (SNRE) for first method (Ito Calculus). The multi-channel blind deconvolution (MBD) technique was applied by using the conventional method [FIR-2 model with two orders & FIR-4 model with 4 orders] and the proposed method [OU model or stochastic calculus]. We compared between these methods by using performance measure SNRE Eq. (3.1) at variant noise. SNRE $_{0.0}$ is the signal to noise ratio of the estimate of Aortic AP with free noise. SNRE$_{0.012}$ is the signal to noise ratio of the estimate with noise variance=0.012, SNRE$_{0.024}$ with noise variance=0.024, etc.

Methods	MAPE $_{0.0}$	MAPE $_{0.012}$	MAPE $_{0.024}$	MAPE $_{0.037}$	MAPE $_{0.049}$	MAPE $_{0.0615}$
FIR-2 order	21.969 %	28.853 %	31.050 %	32.274 %	33.084 %	34.423 %
FIR-4 order	12.941 %	14.061 %	17.120 %	17.966 %	18.090 %	20.801 %
Stochastic Calculus	2.615 %	3.122 %	3.218 %	3.670 %	4.134 %	4.754 %

Table (2): The second performance measure (MAPE) for first method (Ito Calculus). The performance measure for the estimated femoral AP data MAPE (Eq. 3.2) was used to evaluate the cardiovascular system at variant noise for three methods (FIR-2 order & FIR-4 order & Stochastic Calculus]. MAPE$_{0.0}$ is the mean absolute percent error with free noise of the estimated central AP waveform. MAPE$_{0.012}$ is the mean absolute percent error of the estimate with noise variance=0.012, MAPE$_{0.024}$ with noise variance=0.024, etc.

In the first example, it was assumed that the data were noise-free but the order of each of the filters was unknown. In the second example, the data were assumed noisy and the order of each of the filters was unknown. The filter order was estimated using a corrected Akaike information criterion (AIC). In both examples, by using the proposed method, only one coefficient of the two filters was assumed to be unknown and time varying. The rest of the coefficients were time invariant. It was assumed that this coefficient follows an Ornstein-Uhlenbeck (OU) process with some unknown constant parameters. The rest of the filter coefficients were assumed unknown deterministic constants.
In both cases, the proposed approach outperformed the conventional method and, therefore, the method was used in the paper of [18]. The (MAPE) value for the estimated central AP data in this paper was 2.615% see table (2), and the (MAPE) value in the paper of [18] was 3.2%. The pressure at the root of the aorta (central AP) was successfully estimated.

3.1- Example, Noise-Free Measurements

We have two measurements, $y_1(k)$(Femoral AP) and $y_2(k)$ (Radial AP) that are assumed to be represented by the equations:

$$y_1(k) = u(k) + \beta_1 u(k-1) + \beta_2 u(k-2) + \beta_3 u(k-3) + \beta_4 u(k-4) \quad (3.3)$$

$$y_2(k) = u(k) + \alpha_1 u(k-1) + \alpha_2 u(k-2) \quad (3.4)$$

In the conventional method, the order of the filters was assumed to be just two, i.e., β_3=β_4=0. In the proposed method, we made the same order assumption of two but we made one of the coefficients time-varying. Specifically, we assumed that the SDE of $\beta_2(t)$ has the form

$$d\beta_2(t) = c_2(\beta_2(0)\text{-}\beta_2(t))dt + e_2 dW_2(t) \quad (3.5)$$

Where c_2, e_2 are unknown constants that were estimated as explained in Sections 2.2.2 and 2.2.3. Remember that in the conventional method:

$$y_1(k) - y_2(k) = -\alpha_1 y_1(k-1) - \alpha_2 y_1(k-2) + \beta_1 y_2(k-1) + \beta_2 y_2(k-2) \quad (3.6)$$

Figure 5 shows the estimated pressure at the root of the aorta AP from the received signal y_1 (Femoral AP) using the proposed method based on the stochastic calculus (OU).

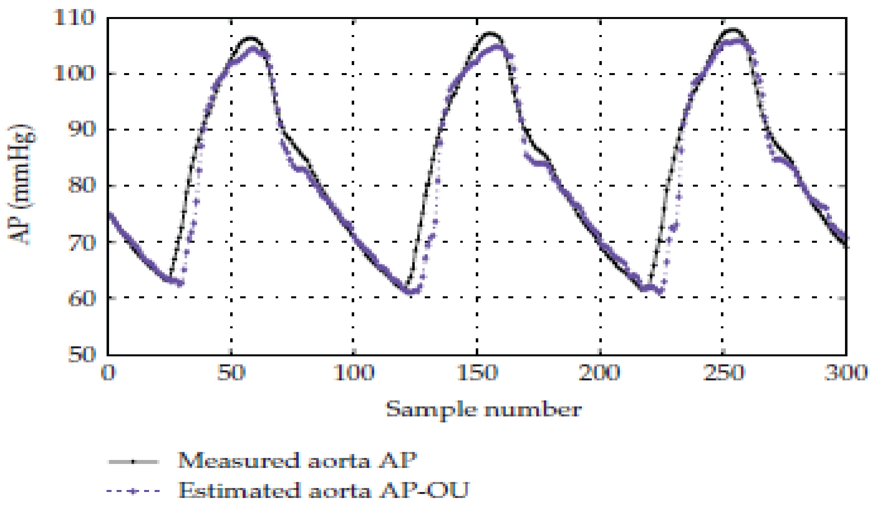


Figure (5): The estimated Aorta AP using OU model and measured Aorta or Central AP waveform

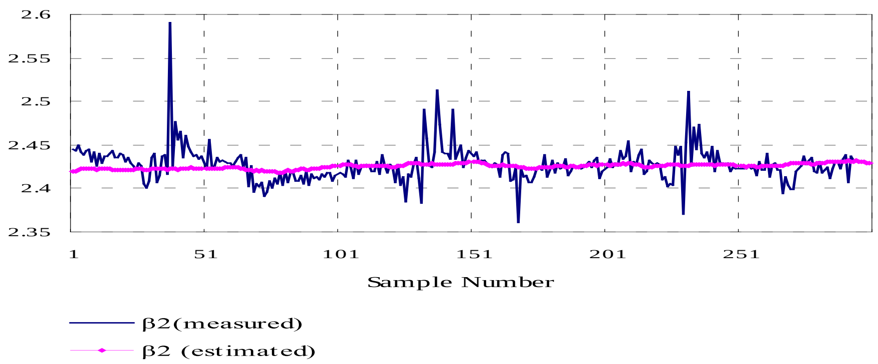


Figure (6): A typical estimate for $\hat{\beta}_2(k)$ compared to the noisy measurements of $\beta_2(k)$.

$SRNE_{0.0}$ = 23.296 db (see Table 1). $SRNE_{0.0}$ is the signal to noise ratio of the estimate with free noise. The mean absolute percent error with free noise ($MAPE_{0.0}$) of the estimated central AP waveform was 2.615%.

We compared the estimated central arterial pressure using the conventional method with second order FIR-2 ($SRNE_{0.0}$ = 9.006 db) and the proposed method OU ($SRNE_{0.0}$ = 23.296 db). We note the difference between them in Figure 7, where we estimated the source signal (Aorta AP) from the received signal (Radial AP). While in Figure 8, after applying the conventional method with fourth-order FIR-4, we obtained a better result than by using FIR-2, because the order of filter is increased and the source signal (Aorta AP) is estimated from the received signal (Femoral AP). Scientifically, Estimation central AP from femoral AP is better than radial AP. But still the stochastic calculus (OU) proved to be the best (see Tables 1 and 2). A typical estimate for $\hat{\beta}_2(k)$ compared to the noisy measurements of $\beta_2(k)$ is shown in Figure 6.

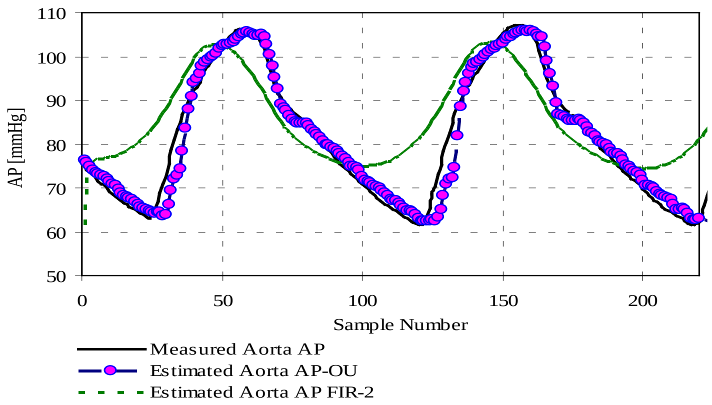
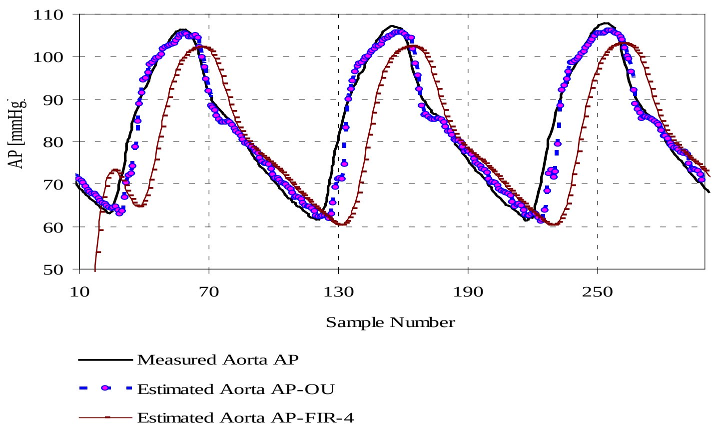


Figure (7): Estimated Aorta AP waveform from radial AP using the conventional method (FIR-2 model with two orders), compared to estimated Aorta AP waveform from femoral AP using stochastic calculus based method (OU).



4. Conclusions

In this report we presented a novel technique to deconvolve the aortic pressure waveform from multiple peripheral artery pressure waveform measurements, using multichannel blind deconvolution. We applied the technique to femoral and radial AP waveforms measured in the swine over 2-minute intervals of a peripheral AP waveform. We assumed that one of the FIR filter coefficients is time varying. Its values were estimated using methods based on the Ito calculus. By this assumption, we were able to compensate for the wrong FIR filter order and the possible time variations of the channels. The results showed superior performance for our proposed approach compared to conventional methods.

In this study, only one unknown time-varying coefficient was assumed to follow the Ornstein-Uhlenbeck stochastic process. Other models could have been used as well. The Ito calculus techniques were used to estimate the coefficients of this Ornstein-Uhlenbeck model. We tested the proposed technique in swine experiments, and our results showed that the MAPE value for the estimated femoral AP data was 2.615%. Our way to reconstructed AP is simple and straightforward. Our method needs only the calculation of pressure wave components in the time domain and does not need calculations in the frequency domain and no need to large computer time. Because of this simplicity, it is quite possible to implement this method in monitoring central pressure AP on line.

In the future, we suggest expanding this method by applying it to real data taken from human cardiovascular simulator. In the presented study, only one parameter was varying with time, while in the future we may use more than just one parameter varying with time. We might study the general case, where no assumptions are imposed on the speed of variations of the time-varying parameters and their numbers. This generalization may improve the accuracy of the estimates. The method of the Malliavin Calculus is proposed to solve this problem [4].

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