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Mechatronic tolerancing: Bond Graph approach

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Abstract: In this work, a new approach has been proposed that allows the analysis and synthesis of mechatronic system tolerances. This approach is based on a statistical method namely Monte Carlo Simulation (MCS). By making a large number of simulations with deviations randomly selected or not in the specified areas, it is possible to perform a statistical analysis of mechatronic system tolerances. To illustrate our approach, we treat the case of the slider crank system driven by a gear motor.

Keywords: Bond graph model; statistical tolerancing; Monte Carlo simulation; slider crank system; gear motor

I. INTRODUCTION

The emergence of new products, such as mechatronic products and new organizational methods such as concurrent engineering, requires the establishment of unified tolerancing support tools between all trades and allowing the simultaneous consideration of design constraints, manufacturing, assembly, interchangeability and quality.

The need to conduct a tolerances analysis of the products in all its life cycle levels led to the emergence of a large number of specific models of the treated domain. Many search and development tracks are then to explore in the multi-domain and multi-physics field of mechatronic systems tolerancing [1,2,3,4].

This paper is organized as follows: Section 2 describes the statistical method used for tolerancing mechatronic systems, namely the Monte Carlo simulation (MCS). Section 3 presents the tolerancing algorithm with the MCS method. The case of study is developed in sections 4, we detailed the approach to model our multiphysics system (Bond Graph approach (BG)) [5], applied the MCS algorithm, then the simulation results are analyzed and interpreted. Finally, section 5 presents our conclusions.

II. MONTE CARLO SIMULATION

Monte Carlo simulation is a simple method for predicting errors manufacturing [6].It consists on generating a large number of experiments in which the numerical output variables are calculated from a set of input variables randomly distributed. For this, the Monte Carlo simulation uses pseudo-random generators with numbers corresponding to different types of statistical distributions. The results obtained are more realistic than those obtained by conventional methods of calculation. The user must define the random distribution of input variables. The number of experiments generated must be large enough to reliably determine the statistical parameters of output variables.

The simulation defines a statistical data generally described by the mean dimension:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} Z_i \qquad (1)$$

and standard deviation:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Z_i - \mu)^2} \qquad (2)$$

where:

- Zi is the resulting dimension to the i_{th} simulation cycle;
- N is the total number of simulation cycles;

For the evaluation of the functional condition realization frequency, the statistical group is converted into a histogram (Figure 1).

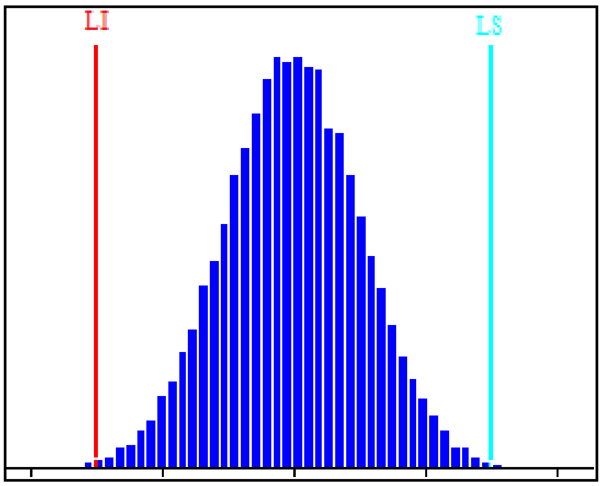


Figure 1: View as histograms (MCS)

The general approach of applying the MCS method is presented in Figure 2.

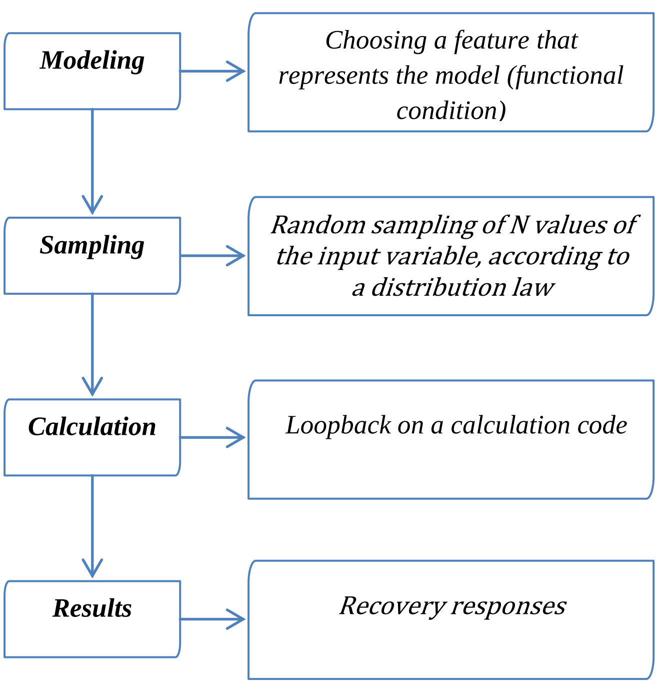


Figure 2: General approach of applying the SMC

The standard deviation σ is a parameter characterizing the dispersion or variation of the values distribution around an average. Higher are the values concentrated around the average, lower is the standard deviation.

In a normal distribution, the standard deviation "σ" is used to establish confidence intervals for desired confidence levels (Figure 3).

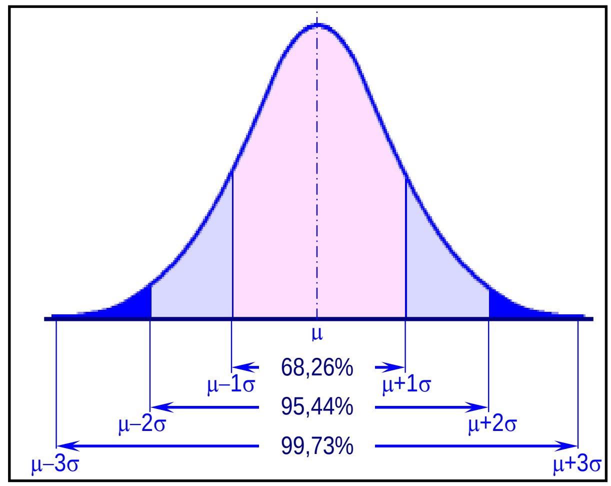


Figure 3: Percentage of trust in an interval

The production process is often considered satisfactory at ±3σ. So 99.73% of assemblies are in the interval $[\mu_J - 3\,\sigma_J\,;\,\mu_J + 3\,\sigma_J]$. For centric distribution, the functional requirement will be respected for 99.73% of assemblies if:

Tolerance on the requirement = 6 σ

III. Mechatronics Tolerancing Algorithm

The parameterization in tolerancing consists in selecting a model and parameters covering in a best way tolerance zones to be studied. It often results in one or more equations of the form $Y = f(x_1, x_2, ..., x_n)$ reflecting the operational condition. This is easily achieved in the case of single-domain tolerancing (geometric, mechanical, electrical, etc.).

However, for complex systems such as mechatronic systems, it is difficult to establish the equations reflecting the functional condition. In effect, these systems fall into the category of hybrid dynamic systems where models with discrete events interacting with those continuous time.

To remedy this problem we combined the Bond Graph method (for modeling mechatronic systems) and the Monte Carlo Simulation method tolerancing. The MCS does not require an explicit expression of the functional condition or an approximation thereof, which is an advantage in this case.

The tolerance analysis is performed according to the algorithm of figure 4.

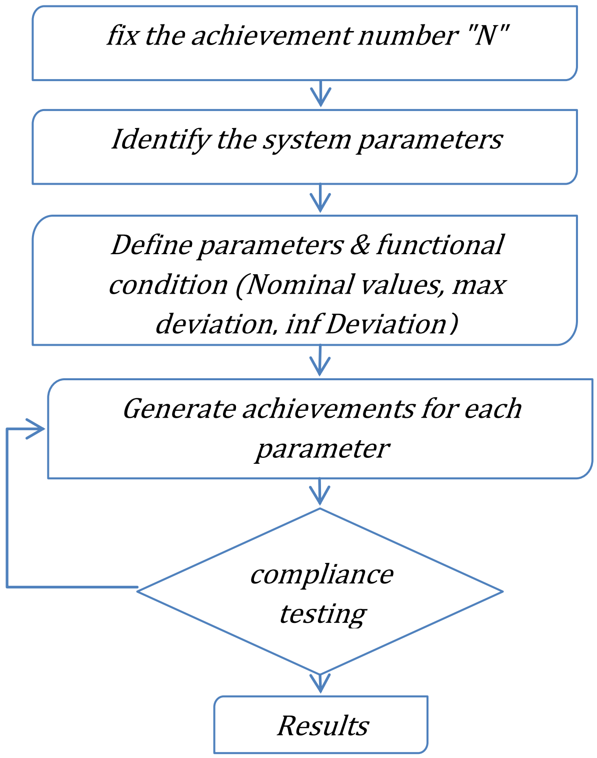


Figure 4: tolerancing algorithm with MCS method

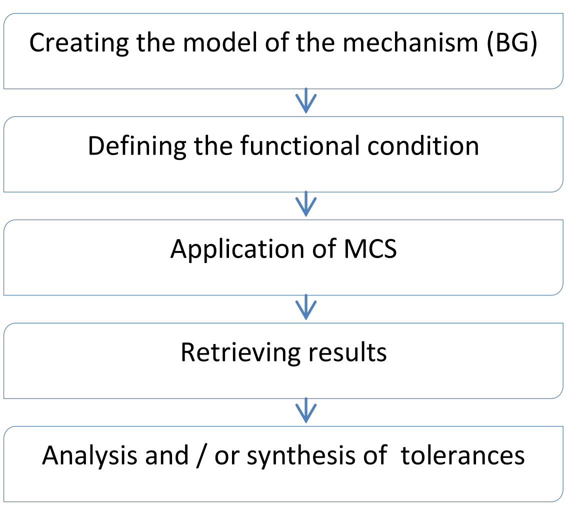


Figure 5: Tolerancing approach

A very large number of models is generated digitally according to a statistical distribution (The normal distribution is often chosen because it corresponds to what actually happens in production). The output values obtained are statistically processed to determine their distribution and statistical parameters.The tolerancing process is then as presented in figure 5.

IV. CASE OF THE STUDY

A. slider crank system Overview : functional condition

In the case of the slider crank system (Figure 6 and 7), the position "X" is given by:

$$X = rcos(\alpha) + \sqrt{L^2 - (rsin(\alpha) + A)^2} \qquad (3)$$

The maximum Xmax position of the piston relative to the crankshaft axis is given by the following equation:

$$X_{max} = \sqrt{(r+L)^2 - A^2} \qquad (4)$$

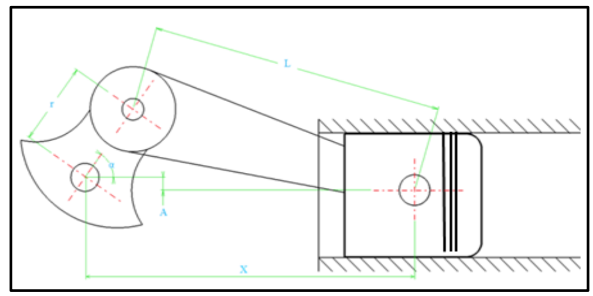
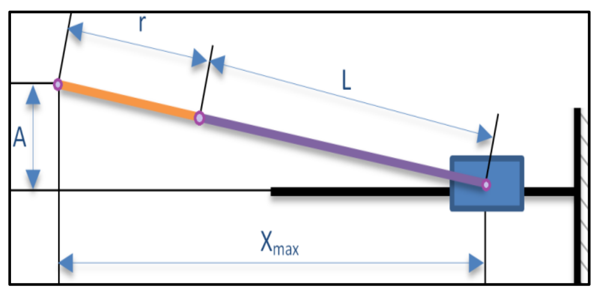


Figure 6: slider crank system

Figure 7: Simplified model of the slider crank system



B. Bond graph Model

The slider crank mechanism is the mechatronic system charge. Bond Graph modeling of that mechanical system requires knowledge the transformation law of the piston rotation in crankshaft translation. The translation speed of the piston is related to the crank rotation speed "ω" by a non-linear law. It is described by the following formula:

$$v = T(\varphi) \, . \, \omega \qquad (5)$$

where:

$$T(\varphi) = r\left[\sin(\varphi) + \frac{\cos(\varphi)(A + r\sin(\varphi))}{\sqrt{L^2 - (A + r\sin(\varphi))^2}}\right] \qquad (6)$$

$$\varphi(t) = \int_0^t \omega(\tau)\, d\tau \qquad (7)$$

This transformation can be modeled by a modulated transformer which module is T (φ).

The bond graph elements of the slider crank are the members (I: J and I: mp) for storing kinetic energy, the element (R: b) energy dispersive (piston friction) and the modulated transformer (MTF: T_PHI) for the passage of the rotation mechanic to the translation mechanic.

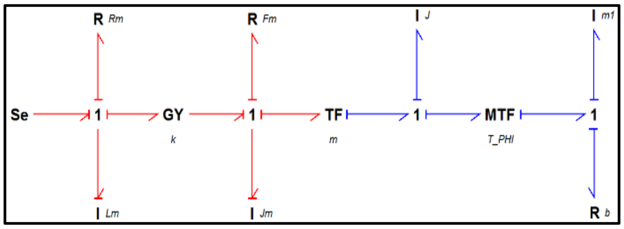


Figure 8: Global bond graph of the mechanism

C. Application of MCS

The Monte Carlo simulation is the most appropriate statistical technique to the problems of tolerancing complex systems. This method is a powerful tool for the tolerances analysis of mechatronic systems where the response function is not necessarily linear.

Moreover, this technique is formally abstract as it is not linked to any constraint and precision may be as important as desired. In particular, the MCS method may give more precise estimates than other methods even if the analytical expression of the response function is not explicitly known. For accurate results, the number of prints N has to be important so not to lead to excessive computation times. Indeed, the accuracy of this statistical analysis increases proportionally as $\sqrt{N}$.

If the value of "N" is sufficiently large, the result of the method reaches a stable value, substantially independent of "N". In fact, if the difference between two results of simulation with different prints is higher than 5%, then the number of prints N is not sufficient to reach the stability.

The result of the MCS method is also a random variable. To characterize the random variable, twenty Monte Carlo simulations are launched for each value of "N" (Figure 8).

These tests determine the extent of the result variations and calculate its standard deviation.Following these tests, we choose "N" equal to 10 000 for the simulations. We chose a distribution of ± 3σ around the mean value for each parameter (Figures 9 and 10).

The nominal parameters and dispersion are given in table I.

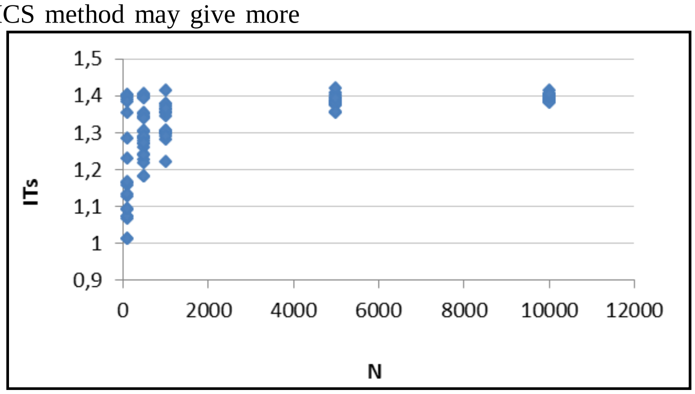


Figure 9: Evolution of estimated values of tolerance intervals versus "N"

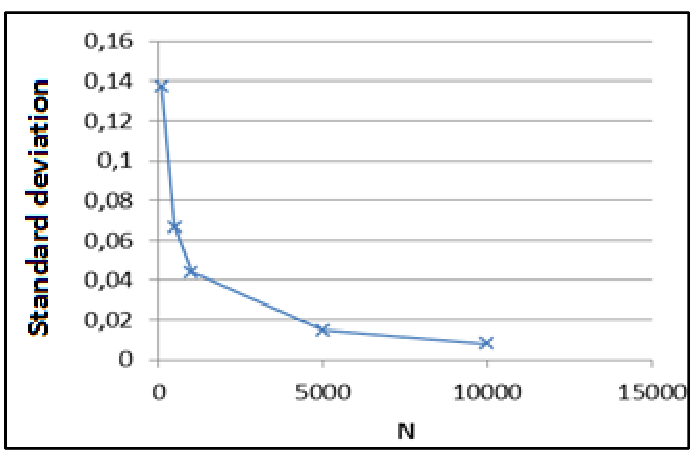


Figure 10: The standard deviation evolution of the estimated tolerance intervals versus "N"

Table I: Nominal parameters and dispersion

Parameters	***Nominal values***	***Limits dimensions***		IT_{MCS}	**σ**
		L_I	L_S		
Jm (kg.m2.10-3)	***0,100***	***0,090***	***0,110***	***±0,010***	***0,0033***
Lm (H.10-3)	***0,630***	***0,567***	***0,693***	***±0,063***	***0,0210***
Rm (Ω)	***1,910***	***1,719***	***2,101***	***±0,191***	***0,0636***
L (mm)	***178***	***177,3***	***178,70***	***±0,700***	***0,2333***
R (mm)	***39***	***38,3***	***39,70***	***±0,700***	***0,2333***
A (mm)	***13***	***12,3***	***13,70***	***±0,700***	***0,2333***

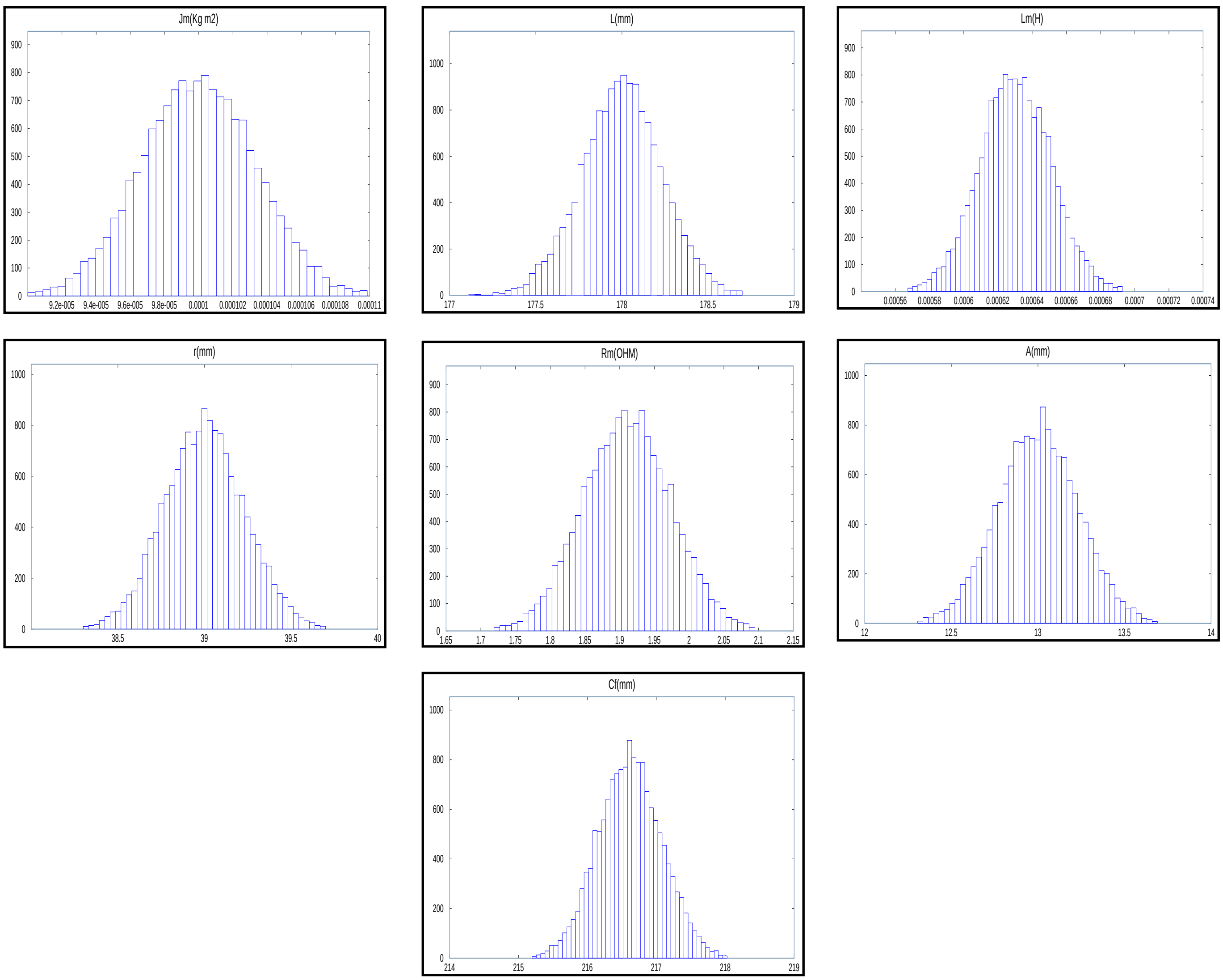


Figure 11: Parameters variation in their tolerance intervals

The histograms relating to the implementation of the MCS method are shown in Figure 11. These histograms show the shape of the distribution generated for each parameter as well as that of the functional condition "Cf".

This will allow us to calculate the statistical parameters and to do a statistical analysis of the functional condition tolerance (table II for mechanical system: slider crank mechanism, and table III for the whole system).

Table II: Mechanical tolerancing with MCS method

Parameters	*Nominal values*	L_i	L_s	IT_{MCS}	σ
L	***178***	***177,300***	***178,000***	***±0,70***	***0,233***
r	***39***	***38,300***	***39,000***	***±0,70***	***0,233***
A	***13***	***12,300***	***13,000***	***±0,70***	***0,233***
Xmax	***216,610249***	***215,41491***	***217,75478***	***±1,16993***	***0,390***

Table III: Mechatronic tolerancing with MCS method

Parameters	*Nominal values*	*Limits dimensions*		IT_{MCS}	σ
		L_I	L_S		
Jm (kg.m2.10-3)	***0,100***	***0,090***	***0,110***	***±0,010***	***0,0033***
Lm (H.10-3)	***0,630***	***0,567***	***0,693***	***±0,063***	***0,0210***
Rm (Ω)	***1,910***	***1,719***	***2,101***	***±0,191***	***0,0636***
L (mm)	***178***	***177,300***	***178,700***	***±0,700***	***0,2333***
R (mm)	***39***	***38,300***	***39,700***	***±0,700***	***0,2333***
A (mm)	***13***	***12,300***	***13,700***	***±0,700***	***0,2333***
Cf	***216.6106***	***215,2466***	***218,0017***	***±1,3775***	***0,4591***

By comparing the MCS method results of the mechanical system to those of mechatronic systems, we find that there is an increase in tolerance zone ($IT_{mcs\_mecan} < IT_{mcs\_mecatr}$) (Table IV). The influence of multi-physical interactions within the mechatronic system has led to offset errors of the components during operation.

Table IV: Comparison of mechanical and mechatronics intervals

Cf	*Li (Cf)*	*Ls (Cf)*	*IT*
IT_{MCS_MECATR}	***215,2466***	***218,0017***	***±1,3775***
IT_{MCS_MECAN}	***215,4149***	***217,7548***	***±1,1699***

Moreover, we find that the statistical tolerance intervals are always contained in the arithmetic tolerances interval ($IT_{mcs\_mecatr} < IT_{pc\_mecatr}$). This allows the designer to expand tolerances intervals and optimize the system operation.

D. Tolerances optimization

The multi-physical nature of mechatronic systems requires the treatment of parameters with non-uniform tolerances distributions and magnitude orders. Indeed, the tolerance interval of each parameter participating in the system operation differs from one component to another and this makes the optimization of tolerances a very daunting task.

Sensitivity analysis [7, 8] is used to quantify the influence of various parameters on the variability of the numerical model response. The sensitivity to a parameter is the variation of the operating condition “Cf” caused by one unit variation of this parameter For example, if “Cf” varies with 0.1 for a parameter variation equal to 0.1, then the sensitivity is one (0.01 / 0.01). Thus, we deduce the most and the least influencing parameters on the model response variability and the parameters who interact. We can act on tolerance for each parameter based on its contribution to the variability of functional condition.

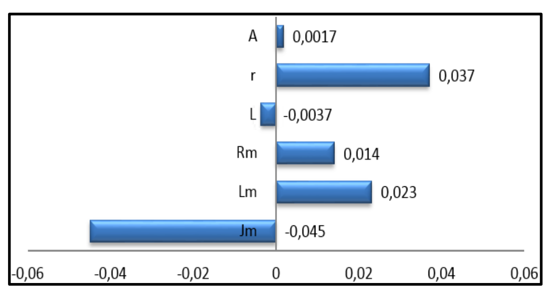


Figure 12 : X_{max} sensitivity to parameter variations in their ITs intervals

Figure 12 shows the sensitivity of the functional condition "Cf" parameters calculated by 20 Sim Software [9].

From this compact representation, we can see the contributions of the various parameters to the operating condition Cf variation .It is important to control the parameters variations whose contribute strongly to the variation of the operational condition "Cf". Conversely; we can affect the parameters tolerances whose influence is low.

We also note that the functional condition “Cf” is:

- Very sensitive to the variation of r and Lm (table V);
- Insensitive to changes in "Lm", "L" and "A". We can then deduce their ITs intervals.

Taking into account the results of the sensitivity analysis, we conducted a progressive expansion of ITs parameters intervals based on their contribution to functional condition "Cf" variations. optimization cycle stops when we have the condition ($IT_{mcs} > IT_{worst\ case}$). For the studied mechanism, the optimization cycle yielded the values shown in the table VI.

This information can be used by the designer to make technical decisions regarding the optimization, the performance and cost of the mechatronic system design.

Table V: influence of a 0,02 variation of the parameter r on Cf

Parameters	Nominal	Dimensions limits		IT_{MCS_MECATR}	σ
		L_I	L_S		
Jm (kg.m2).10-3	0,100	0,090	0,110	±0,010	0,0033
Lm (H).10-3	0,630	0,567	0,693	±0,063	0,0210
Rm (Ω)	1,910	1,719	2,101	±0,191	0,0636
L (mm)	178	177,300	178,700	±0,700	0,2333
r (mm)	39	38,280	39,720	± 0,720	0,2400
A (mm)	13	12,300	13,700	± 0,700	0,2333
Cf	216.6106	215,1668	218,0365	± 1,4348	0,4783

Table VI: Its Intervals optimized with MCS method

Parameters	Nominal	Dimensions limits		IT_{opti}	σ
		L_I	L_S		
Jm (kg.m2).10-3	0,100	0,090	0,110	±0,010	0,0033
Lm (H).10-3	0,630	0,550	0,710	±0,080	0,026
Rm (Ω)	1,910	1,713	2,107	±0,197	0,066
L (mm)	178	176,50	179,500	±1,500	0,50
r (mm)	39	38,300	39,700	±0,700	0,2333
A (mm)	13	11,500	14,500	±1,500	0,50
Cf	216.6106	215,2500	218,0129	±1,42683	0,4605

V. CONCLUSION

This work contributes to the modeling and control of variability in mechatronics product design phase. It is particularly the control of variations in several parameters of different physical domains and their effects on the kinematics of the system studied.

The tolerance analysis is not a common operation for mechatronic systems; we have shown that such an analysis is possible. We applied our approach to the crank slider system driven by a gearmotor, but it is applicable to other mechanisms.

The flexibility of the bond graph language model for different physical domains and using a method combining analysis "worst case" and Monte Carlo simulation allow the analysis and optimization of the tolerances intervals in the mechatronic system case.

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