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Application of two mixed Quadrature rules using an anti-Gaussian Quadrature rule in the Adaptive quadrature routine

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Abstract: A model is set up which embodies the basic features of Adaptive quadrature routines involving mixed rules. Not before mixed quadrature rules basing on anti-Gaussian quadrature rule have been used for fixing termination criterionin Adaptive quadrature routines. Two mixed quadrature rules of higher precision for approximate evaluation of real definite integrals have been constructed using an anti-Gaussian rule for this purpose. The first is linear combination of anti-Gaussian three point rule and Fejers three point first rule, the second is the linear combination of anti-Gaussian three point rule and Fejers three point second rule.The analytical convergence of the rules have been studied. The error bounds have been determined asymptotically. Adaptive quadrature routines being recursive by nature, a termination criterion is formed taking in to account two mixed quadrature rules. The algorithm presented in this paper has been "C" programmed and successfully tested on different integrals. The efficiency of the process is reflected in the table at the end.

Keywords:Gauss Legendre two point rule, anti-Gaussian rule, Fejers three point first rule, Fejers three point second rule, mixed quadrature rule ,Adaptive quadrature.

2000 Mathematics Subject Classification: 65D30, 65D32

1. Introduction

Given a real integrable function f an interval $[a,b]$ and a prescribed tolerance ε, it is desired to compute an approximation P to the integral $I=\int_a^b f(x)dx$, So that $|P-I|\leq\varepsilon$.This can be done following adaptive integration schemes developed in papers [2,3,6-9,11]. In adaptive integration, the points at which the integrand is evaluated are chosen in a way that depends on the nature of the integrand. The basic principle of adaptive quadrature routines is discussed in the following manner.

If c is any point between a and b then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

The idea is that if we can approximate each of the two integrals on the right to within a specified tolerance, then the sum gives us the desired result. If not we can recursively apply the Adaptive property to each of the intervals $[a,c]$ and $[c,b]$. Adaptive subdivision of course has geometrical appeal. It seems intuitive that points should be concentrated in regions where the integrand is badly behaved. The whole interval rules can take no direct account of this.

In this paper we design an algorithm for numerical computation of integrals in the Adaptive quadrature routines involving mixed rules. The literature of the mixed quadrature rule [5-9,11] involves construction of a symmetric quadrature rule of higher precision as a linear/convex combination of two other rules of equal lower precision.

About anti-Gaussian quadrature:

Dirk P. Laurie [1] is first to coin the idea of anti-Gaussian quadrature formula . An anti-Gaussian quadrature formula is an $(n+1)$ point formula of degree $(2n-1)$ which integrates all polynominals of degree upto $(2n+1)$ with an error equal in magnitude but opposite in sign to that of n-point Gaussian formula.

If $H^{(n+1)}(f) = \sum_{i=1}^{n+1} \lambda_i \; f\,(\xi_i)$ be $(n+1)$ point anti-Gaussian formula and $G^{(n)}(p)$ be n point Gaussian formula then by hypothesis .

$I(p) - H^{(n+1)}(p)$ = - $(I(p) - G^{(n)}(p)), p \in P_{2n+1}$ where p is a polynomial of degree $\leq (2n+1)$. In this paper we design a three point anti-Gaussian rule following LAURIE [1].

As the anti-Gaussian three point rule $RH_w^3(f)$ [1] and Fejers three point first rule $Rfj_1(f)$ rules are of same precision (i.e precision 3), one can form a mixed quadrature rule $RH_w^3 fj_1(f)$ of precision five by taking the linear combination of these two rules. Similarly one can form a mixed quadrature rule $RH_w^3 fj_2(f)$ of precision five by taking the linear combination of the anti-Gaussian three point rule $RH_w^3(f)$ and Fejers three point second rule $Rfj_2(f)$ rules are of same precision (i.e precision 3)

So far no body has used anti-Gaussian three point rule in mixed quadrature, first time in this paper we incorporate the idea of anti-Gaussian three point rule to form two mixed quadrature rules in Adaptive quadrature routines.

To prepare an algorithm for Adaptive quadrature routines in evaluating an integral $I = \int_a^b f(x)dx$,we use the following two mixed quadrature rules.

(i) $RH_w^3 fj_1(f)$ as I_1

(ii) $RH_w^3 fj_2(f)$ as I_2

2. A Simple Adaptive Strategy

The input to these schemes is $a, b, \varepsilon, n, f,$ the output $I \approx \int_a^b f(x)dx$ with the error hopefully less than $\varepsilon; n$ is the number of intervals initially chosen. A Simple adaptive strategy is out lined in the following step algorithm.

Step - 1 : An approximation I_1 to $I \approx \int_a^b f(x)dx$ is computed.

Step - 2 : The interval is divided into pieces $[a,c]$ and $[c,b]$.

Where $c = \frac{a+b}{2}$ and then $I_2 = \int_a^c f(x)dx$ and

$I_3 = \int_c^b f(x)dx$ are computed.

Step - 3 : $I_2 + I_3$ is computed with I_1 to estimate the error in $I_2 + I_3$.

Step - 4 : If | estimated error $| \leq \varepsilon/2$ (termination-criterion), then $I_2 + I_3$ is accepted as an approximation to $I \approx \int_a^b f(x)dx$. Otherwise the same procedure is applied to $[a,c]$ and $[c,b]$ allowing each piece a tolerance of $\varepsilon/2$.

Adaptive quadrature routines essentially consist of applying the rules $RH_w^3 fj_1(f)$ and $RH_w^3 fj_2(f)$ to each of the sub intervals covering $[a,b]$ until the termination criterion is satisfied. If the termination criterion is not satisfied on one or more the sub intervals, then those subintervals must be further sub divided and the entire process repeated.

3. Construction of anti-Gaussian three point rule from Gauss Legendre two point rule

We choose the Gauss-Legendre two point rule ,

$$G_w^2(f) = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right) \qquad (3.1)$$

We develop a three point anti-Gaussian rule $H_w^3(f)$ from two point Gaussian rule $G_w^2(f)$.

Using the principle $I(p) - H_w^3(f) = -(I(p) - G_w^3(f))$ as adopted in Laurie [1], after simplification we get

$$H_w^3(f) = 2\int_{-1}^{1} f(x)dx - (G_w^2(f)). \qquad (3.2)$$

$$\Rightarrow \alpha_1 f(\xi_1) + \alpha_2 f(\xi_2) + \alpha_3 f(\xi_3) = 2\int_{-1}^{1} f(x)dx - (G_w^2(f)), \qquad (3.3)$$

Taking $H_w^3(f) = \alpha_1 f(\xi_1) + \alpha_2 f(\xi_2) + \alpha_3 f(\xi_3)$

- In order to obtain the unknown weights and nodes, we assume that

(a) The rule is exact for all polynomial of degree ≤ 3 .

(b) The rule integrates all polynomials of degree up to five with an error equal in magnitude and opposite in sign to that of Gaussian rule. Thus we obtain following system of six equations having six unknowns namely $\alpha_i, \xi_i \quad (i = 1,2,3)$

Solving the above system of equation we get,

$$\alpha_1 = \frac{5}{13} = \alpha_3, \alpha_2 = \frac{16}{13}, \xi_1 = \pm\sqrt{\frac{13}{15}}, \xi_2 = 0, \xi_3 = \mp\sqrt{\frac{13}{15}}$$

Hence, the method becomes.

$$\int_{-1}^{1} f(x)dx \cong RH_w^3(f) = \frac{5}{13} f\left(-\sqrt{\frac{13}{15}}\right) + \frac{16}{13} f(0) + \frac{5}{13} f\left(\sqrt{\frac{13}{15}}\right) \qquad (3.4)$$

The error associated with the method (3.4) is computed as

$$EH_w^3(f) = \int_{-1}^{1} f(x)dx - RH_w^3(f) = \frac{-f^{iv}(0)}{135} - \frac{1016 f^{vi}(0)}{7! \times 675} \ldots \qquad (3.5)$$

4. Construction of mixed quadrature rule by using anti-Gaussian three point rule with Fejers three point first rule

We have the anti-Gaussian three point rule,

$$RH_w^3(f) = \frac{5}{13} f\left(-\sqrt{\frac{13}{15}}\right) + \frac{16}{13} f(0) + \frac{5}{13} f\left(\sqrt{\frac{13}{15}}\right) \qquad (4.1)$$

Then the Fejers three point second rule:

$$Rfj_1(f) = \frac{1}{9}[4\{f(\frac{\sqrt{3}}{2}) + f(-\frac{\sqrt{3}}{2})\} + 10 f(0)] \qquad (4.2)$$

Each of the rules $RH_w^3(f)$ and $Rfj_1(f)$ is of precision three.

Let $EH_w^3(f)$ and $Efj_1(f)$ denotes the error in approximating the integrals $I(f)$ by the rules $RH_w^3(f)$ and $Rfj_1(f)$ respectively.

Now $I(f) = RH_w^3(f) + EH_w^3(f) \qquad (4.3)$

$I(f) = Rfj_1(f) + Efj_1(f) \qquad (4.4)$

Using Maclaurines expansion of function in equation (4.1) and (4.2). We have

$$EH_w^3(f) = \frac{-f^{iv}(0)}{135} - \frac{2\times 508}{7!\times 675} f^{vi}(0) \quad (4.5)$$

$$Efj_1(f) = -\frac{f^{iv}(0)}{240} - \frac{f^{vi}(0)}{8064} \quad (4.6)$$

Eliminating $f^{iv}(0)$ from equation (4.5) and (4.6) we have

$$I(f) = \frac{1}{7}[16Rfj_1(f) - 9RH_w^3(f)] + \frac{1}{7}[16Efj_1(f) - 9EH_w^3(f)] \quad (4.7)$$

or $$I(f) = RH_w^3 fj_1(f) + EH_w^3 fj_1(f) \quad (4.8)$$

Where $EH_w^3 fj_1 = \frac{1}{7}[16Efj_1(f) - 9EH_w^3(f)]$ and

$$RH_w^3 fj_1 = \frac{1}{7}[16Rfj_1(f) - 9RH_w^3(f)] \quad (4.9)$$

$$\Rightarrow RH_w^3 fj_1(f) = \frac{64}{63}\{f(\frac{\sqrt{3}}{2}) + f(-\frac{\sqrt{3}}{2})\} + \frac{160}{163} f(0) - \frac{45}{91}\{f(\sqrt{\frac{13}{15}}) + f(-\sqrt{\frac{13}{15}})\} - \frac{144}{91} f(0) \quad (4.10)$$

This is the desired mixed Quadrature rule of precision five. for the approximate evaluation of $I(f)$. The truncation error generated in this approximation is given by.

$$EH_w^3 fj_1(f) = \frac{1}{7}[16Efj_1(f) - 9EH_w^3(f)] \quad (4.11)$$

or $$EH_w^3 fj_1(f) = \frac{941}{264600} f^{vi}(0) + \ldots \quad (4.12)$$

$$\left|EH_w^3 fj_1(f)\right| \cong \frac{941}{264600}\left|f^{vi}(0)\right| \quad (4.13)$$

The rule $RH_w^3 fj_1(f)$ is called a mixed type rule of precision five as it is constructed from two different types of the rules of the same precision .

5. Construction of mixed Quadrature rule by using anti-Gaussian three point rule with Fejers three point second rule

We have the anti-Gaussian three point rule

$$RH_w^3(f) = \frac{5}{13} f\left(-\sqrt{\frac{13}{15}}\right) + \frac{16}{13} f(0) + \frac{5}{13} f\left(\sqrt{\frac{13}{15}}\right) \quad (5.1)$$

and Fejers three point second rule.

$$Rfj_2(f) = \frac{2}{3}\left[f(-\frac{1}{\sqrt{2}}) + f(\frac{1}{\sqrt{2}}) + f(0) \right] \quad (5.2)$$

Each of the rule $RH_w^3(f)$ and $Rfj_2(f)$ is of precision three.

Let $EH_w^3(f)$ and $Efj_2(f)$ denote the error in the approximating the integrals $I(f)$ by the rules $RH_w^3(f)$ and $Rfj_2(f)$ respectively.

Now $I(f) = RH_w^3(f) + EH_w^3(f)$ (5.3)

$$I(f) = Rfj_2(f) + Efj_2(f) \quad (5.4)$$

Using Maclaurine's expansion of function in equation (5.1) and (5.2) we have,

$$Efj_2(f) = \frac{f^{iv}(0)}{360} + \frac{f^{vi}(0)}{6048}..... \quad (5.5)$$

$$EH_w^3(f) = -\frac{f^{iv}(0)}{135} - \frac{1016 \times f^{vi}(0)}{7! \times 675}... \quad (5.6)$$

Eliminating $f^{iv}(0)$ from (5.5) and (5.6), we have

$$I(f) = \frac{1}{11}[3RH_w^3(f) + 8Rfj_2(f)] + \frac{1}{11}[3EH_w^3(f) + 8Efj_2(f)] \quad (5.7)$$

$$I(f) = RH_w^3 fj_2(f) + EH_w^3 fj_2(f) \quad (5.8)$$

Where $RH_w^3 fj_2(f) = \frac{1}{11}[3RH_w^3(f) + 8Rfj_2(f)]$ (5.9)

$$RH_w^3 fJ_2(f) = \frac{1}{11}[\frac{15}{13} f(-\sqrt{\frac{13}{15}}) + \frac{48}{13} f(0) + \frac{15}{13} f(\sqrt{\frac{13}{15}}) + \frac{16}{3} f(-\frac{1}{\sqrt{2}}) + \frac{16}{3} f(\frac{1}{\sqrt{2}}) + \frac{16}{3} f(0)]$$

(5.10)

This is the desired mixed quadrature rule of precision five for the approximate evaluation of $I(f)$. The truncation error generated in this approximation is given by.

$$EH_w^3 fj_2(f) = \frac{1}{11}[3EH_w^3(f) + 8Efj_2(f)]$$

(5.11)

or $$EH_w^3 fj_2(f) = \left(\frac{3213}{89100 \times 371}\right) f^{vi}(0) + ... \quad (5.12)$$

$$|\ EH_w^3 fj_2(f)\ | \approx |\ \left(\frac{3213}{89100 \times 371}\right) f^{vi}(0)\ |$$

The rule $RH_w^3 fj_2(f)$ is called a mixed type rule of precision five as it is constructed from two different types of the rules of the same precision .

6. Error analysis

An asymptotic error estimate and an error bound of the rule (4.9) and (4.11) are given by.

Theorem - 6.1

Let $f(x)$ be sufficiently differentiable function in the closed interval $[-1,1]$.Then the error $EH_w^3 fj_1(f)$ associated with the rule $RH_w^3 fj_1(f)$ is given by

$$\left|EH_w^3 fj_1(f)\right| \le \frac{941}{264600}\left|f^{vi}(\eta)\right|, \eta_{1,}\eta_2 \in [-1,1] \qquad (6.1.1)$$

Proof :

From (4.9) and (4.11) we have

$$RH_w^3 fj_1(f) = \frac{1}{7}[16Rfj_1(f) - 9RH_w^3(f)]$$

And the truncation error generated in this approximation is given by

$$EH_w^3 fj_1(f) = \frac{1}{7}16[Efj_1(f) - 9EH_w^3(f)] = \frac{941}{264600}\left|f^{vi}(0)\right| + ...$$

Hence we have $\left|EH_w^3 fj_1(f)\right| \approx \frac{941}{264600}\left|f^{vi}(\eta)\right|$

Theorem- 6.2

The bound of the truncation error

$EH_w^3 fj_1(f) = I(f) - RH_w^3 fj_1(f)$ is given by

$$\left|EH_w^3 fj_1(f)\right| \le \frac{M}{105}\left|\eta_2 - \eta_1\right|, \eta_1, \eta_2 \in [-1,1] \qquad (6.2.1)$$

where $M = \max_{-1 \le x \le 1}\left|f^{v}(x)\right|$

Proof : We have $EH_w^3(f) = -\frac{1}{135} f^{iv}(\eta_1)$ and

$$Efj_1(f) = -\frac{1}{240} f^{iv}(\eta_2)$$

$$EH_w^3 fj_1(f) = \frac{1}{7}[16Efj_1(f) - 9EH_w^3(f)]$$

$$\left|EH_w^3 fj_1(f)\right| \le \frac{1}{105}\left|f^{iv}(\eta_2) - f^{iv}(\eta_1)\right|$$

$$= \frac{1}{105}\int_{\eta_1}^{\eta_2} f^{v}(x)dx, \text{ where } \eta_1,\eta_2 \in [-1,1] .$$

$$\le \frac{M}{105}\left|\eta_2 - \eta_1\right|$$

Where $M = \max_{-1 \le x \le 1} f^{v}(x)$

Which gives a theoretical error bound as η_1,η_2 are unknown points in $[-1,1]$. From this Theorem it is clear that the error in approximation will be less if points η_1,η_2 are closer to each other.

Corollary - 6.3

The error bound for the truncation error $EH_w^3 fj_1(f)$ is given by

$$\left|EH_w^3 fj_1(f)\right| \le \frac{2M}{105} \qquad (6.3.1)$$

Proof :

The proof follows from Theorem (6.2) and $\left|\eta_1 - \eta_2\right| \le 2$.

Theorem - 6.4

Let $f(x)$ be sufficiently differentiable function in the closed interval $[-1,1]$ .Then the error .

$EH_w^3 fj_2(f)$ associated with the rule $RH_w^3 fj_2(f)$ is given by

$$\left|EH_w^3 fj_2(f)\right| \le \frac{3213}{89100 \times 371} \mid f^{vi}(\eta) \mid, \eta_1, \eta_2 \in [-1,1]$$

(6.4.1)

Proof : Similar to the proof of Theorem (6.1)

Theorem - 6.5

The bound of the truncation error

$EH_w^3 fj_2(f) = I(f) - RH_w^3 fj_2(f)$ is given by

$$\left|EH_w^3 fj_2(f)\right| \le \frac{M}{495}\left|\eta_2 - \eta_1\right|, \text{ Where } \eta_1,\eta_2 \in [-1,1] .$$

where $M = \max_{-1 \le x \le 1} f^{v}(x)$

Proof : Similar to the proof of Theorem (6.2)

Corollary - 6.6

The error bound for the truncation error $EH_w^3 fj_2(f)$ is given by

$$\left|EH_w^3 fj_2(f)\right| \le \frac{2M}{495} \qquad (6.6.1)$$

Proof : The proof follows from Theorem (6.5) $|\eta_1 - \eta_2| \le 2$.

7. Numerical verification

Comparison among the rule:

$(RGL_2(f), RH_w^3(f), Rfj_1(f), Rfj_2(f), RH_w^3 fj_1(f), RH_w^3 fj_2(f))$

Table-7.1: Approximation of the integrals in the whole interval quadrature routine

Integrals	Exact value(I)	Approximate Value					
		$RGL_2(f)$	$RH_w^3(f)$	$Rfj_1(f)$	$Rfj_2(f)$	$RH_w^3 fj_1(f)$	$RH_w^3 fj_2(f)$
$I_1 = \int_{-1}^{1} e^x dx$	**2.35040238**	**2.34269608**	**2.35811374**	**2.35469453**	**2.34745578**	**2.3502984**	**2.3503625**
$I = \int_0^1 e^{-x^2} dx$	**0.746825**	**0.746594**	**0.747054**	**0.74694700**	**0.74672971**	**0.74680939**	**0.74681816**
$I = \int_0^1 e^{x^2} dx$	**1.4627**	**1.4541678**	**1.4711569**	**1.46726574**	**1.45926153**	**1.4622627**	**1.46250574**
$I_4 = \int_1^3 \left(\frac{\sin^2 x}{x}\right) dx$	**0.7948251**	**0.79856801**	**0.7911007**	**0.79261229**	**0.79607516**	**0.79455569**	**0.7947185**
$I_5 = \int_0^1 \sqrt{x} dx$	**0.6666666**	**0.67688733**	**0.65983410**	**0.6650026**	**0.67122324**	**0.7341392**	**0.6681171**

Table-7.2: Approximation of the integrals in the Adaptive quadrature routines

Integrals	Exact Value(I)	Approximate Value $RGL_2(f)$	No of step	Error	Approximate Value $RH_w^3(f)$	No of step	Error	Prescribed Tolerance
$I_1 = \int_{-1}^{1} e^x dx$	**2.350402387**	**2.350402386**	**07**	**0.0000000004**	**2.350402488**	**17**	**0.0000001**	**0.00001**
$I = \int_{0}^{1} e^{-x^2} dx$	**0.746825**	**0.74682412**	**15**	**0.0000008**	**0.746824138**	**15**	**0.00000086**	**0.00001**
$I_3 = \int_{0}^{1} e^{x^2} dx$	**1.4627**	**1.46265166**	**19**	**0.000048**	**1.46261518**	**19**	**0.000048**	**0.00001**
$I_4 = \int_{1}^{3} \left(\frac{\sin^2 x}{x} \right) dx$	**0.7948251**	**0.7948251**	**13**	**0**	**0.79482539**	**13**	**0.00000012**	**0.00001**
$I_5 = \int_{0}^{1} \sqrt{x} dx$	**0.79487251**	**0.666668**	**19**	**0.0000022**	**0.66666457**	**19**	**0.000002**	**0.00001**

Table-7.3: Approximation of the integrals in the Adaptive quadrature routines

Integrals	Exact Value(I)	Approximate Value $Rfj_1(f)$	No of step	Error	Approximate Value $Rfj_2(f)$	No of step	Error	Prescribed Tolerance
$I_1 = \int_{-1}^{1} e^x dx$	**2.35040238**	**2.35040307**	**09**	**0.0000006**	**2.3504019**	**09**	**0.00000045**	**0.00001**
$I = \int_{0}^{1} e^{-x^2} dx$	**0.746825**	**0.74682417**	**07**	**0.0000008**	**0.7468241**	**07**	**0.0000008**	**0.00001**
$I_3 = \int_{0}^{1} e^{x^2} dx$	**1.4627**	**1.4626521**	**11**	**0.000047**	**1.4626512**	**09**	**0.000048**	**0.00001**
$I_4 = \int_{1}^{3} \left(\frac{\sin^2 x}{x} \right) dx$	**0.7948251**	**0.7948253**	**13**	**0.0000002**	**0.7948253**	**11**	**0.0000002**	**0.00001**
$I_5 = \int_{0}^{1} \sqrt{x} dx$	**0.666666**	**0.6666649**	**15**	**0.000001**	**0.666668**	**17**	**0.0000021**	**0.00001**

Table-7.4: Approximation of the integrals in the Adaptive quadrature routines:

Integrals	Exact Value(I)	Approximate Value $RH_w^3 fj_1(f)$	No of step	Error	Approximate Value $RH_w^3 fj_2(f)$	No of step	Error	Prescribed Tolerance
$I_1 = \int_{-1}^{1} e^x dx$	2.350402387	2.350402358	03	0.00000002	2.350402376	03	0.00000003	0.00001
$I = \int_{0}^{1} e^{-x^2} dx$	0.746825	0.7468241	03	0.0000008	0.7468241	03	0.0000008	0.00001
$I_3 = \int_{0}^{1} e^{x^2} dx$	1.4627	1.46265172	05	0.000048	1.4626516	03	0.000048	0.00001
$I_4 = \int_{1}^{3} \left(\frac{\sin^2 x}{x} \right) dx$	0.7948251	0.7948251	03	0.00000006	0.7948251	03	0.00000006	0.00001
$I_5 = \int_{0}^{1} \sqrt{x} dx$	0.6666666	0.666675	13	0.000009	0.666667	13	0.000001	0.00001

8. Observation

From the table (7.1) it is observed that the results obtained due to the mixed rules ($RH_w^3 fj_1(f), RH_w^3 fj_2(f)$) are better than its constituent rules ($RGL_2, RH_w^3(f), Rfj_1(f), Rfj_2(f)$) when applied on whole interval. However when these rules are used in Adaptive mode, tables (7.2,7.3,7.4) depict that the mixed quadrature rules $RH_w^3 fj_1(f)$ and $RH_w^3 fj_2(f)$ using anti-Gaussian 3 point rule ($RH_w^3(f)$) give very good result and less number of steps than its constituent rules $RGL_2(f), RH_w^3(f)$, $Rfj_1(f)$, $Rfj_2(f)$ when tested on a number of integrals. Even the results are better than the results of previously solved papers [7-9,11] .

9. Conclusion

After observation one can smartly draw conclusion over the efficiency of the two rules formed in this paper as follows

(i) First mixed rule ($RH_w^3 fj_1(f)$) is more efficient than its constituent rules $RGL_2(f), RH_w^3(f)$, $Rfj_1(f)$ and previously developed mixed rules.

(ii) Second mixed rule $RH_w^3 fj_2(f)$ is more efficient than its constituent rules $RGL_2(f), RH_w^3(f), Rfj_2(f)$ and previously developed mixed rules.

In this paper we have concentrated mainly on computation of definite integrals in the Adaptive quadrature routines involving mixed quadrature rules. We observed that mixed quadrature rules so formed can be very well used for evaluating real definite integrals in the Adaptive quadrature routine.

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