

Reducing PAPR of OFDM Using PTS Techniques for Different Number of Sub-blocks

Md. Ibrahim Abdullah¹, Sajib Kumar Kundu², Md. Shamim Hossain³, Md. Zulfikar Mahmud⁴

^{1,2,3}Computer Science and Engineering, Islamic University, Kushtia, Bangladesh

⁴Computer Science and Engineering, PSTU, Pabna, Bangladesh

Abstract: Orthogonal Frequency Division Multiplexing (OFDM) technology promises to be a key technique for achieving the high data capacity and spectral efficiency requirements for wireless communication system of the near future. Peak to average power ratio (PAPR) is a major drawback of multicarrier transmission system which leads to reduce the efficiency of Radio Frequency (RF) amplifiers and increase the complexity in the analog to digital and digital to analog conversion. In this paper, we present PAPR reduction on Partial Transmit Sequences (PTS) method using different number of sub-blocks. Simulation result shows that the PAPR reduces when the numbers of sub-blocks are increases.

Keywords: OFDM, PAPR, PTS, CCDF, SUB-BLOCKING.

I. INTRODUCTION

OFDM is a multicarrier modulation technique which seems to be an attractive candidate for fourth generation (4G) wireless communication systems. OFDM offer high spectral efficiency, immune to the multipath delay, low inter-symbol interference (ISI), immunity to frequency selective fading and high power efficiency [1]. Due to these merits OFDM is chosen as high data rate communication systems such as Digital Video Broadcasting (DVB) and based mobile worldwide interoperability for microwave access (mobile Wi-MAX). OFDM faces several challenges. The key challenges are large peak to average power ratio (PAPR) due to nonlinearity of amplifier, phase noise problems of local oscillator, frequency offset due to Doppler shift or difference between transmitter and receiver.

In OFDM system output is superposition of multiple sub-carriers. In this case some instantaneous power output might increase greatly and become far higher than the mean power of system. To transmit signals with such high PAPR, it requires power amplifiers with very high power scope. These kinds of amplifiers are very expensive and have low efficiency-cost. If the peak power is too high, it could be out of the scope of the linear power amplifier. This gives rise to non-linear distortion which changes the superposition of the signal spectrum resulting in performance degradation.

Several PAPR reduction techniques have been proposed by researchers. PAPR reduction schemes can be classified according to several criteria. These techniques are divided into two groups [2]. There are signal scrambling techniques and signal distortion

techniques. Block coding techniques [3], selected mapping (SLM) [4][5], partial transmit sequence (PTS) [6] etc. are signal scrambling techniques. Signal distortion techniques are clipping and filtering [7], peak windowing [8], envelope scaling [9], peak reduction carrier [10]. On the other hand, the PAPR reduction schemes can be also categorized according to whether they are deterministic or probabilistic. Deterministic schemes, such as clipping and peak canceling, strictly limit the PAPR of the OFDM signals below a given threshold level. Probabilistic schemes, however, statistically improve the characteristics of the PAPR distribution of the OFDM signals avoiding signal distortion. SLM and PTS are examples of the probabilistic scheme because several candidate signals are generated and that which has the minimum PAPR is selected for transmission. In this paper, we have studied the PTS technique to study its performance in PAPR reduction.

The paper is organized as follows: first we investigate the PAPR in OFDM systems in section 2 and then we describe the PTS technique in section 3. Simulation results are presented in section 4 and finally we present some conclusions in section 5.

II. PAPR OF OFDM

An OFDM symbol is made of sub-carriers modulated by constellations mapping. OFDM symbols can be given as the sum of a numbers of independent symbols which are modulated onto sub channels of equal bandwidth. Let X_k ($k = 0, 1, \dots, N-1$) denote the input data symbol whose period is T . Then the complex representation of an OFDM symbol is given as:

$$x(t) = \sum_{k=0}^{N-1} X_k \cdot e^{j2\pi k \Delta f t}, 0 \leq t \leq NT \quad (1)$$

Where N is the number of subcarriers, and $\Delta f = 1/NT$ is the subcarrier spacing. The samples are denoted by x_n ($n = 0, 1, \dots, LN - 1$) for the OFDM symbols with the sampling rate L .

Theoretically, large peaks in OFDM system can be expressed as Peak-to-Average Power Ratio, or referred to as PAPR. It is usually defined as [9]:

$$PAPR[x(t)] = \frac{P_{peak}}{P_{average}} = 10 \log_{10} \frac{\max [X(n)]^2}{E[x_n]^2} \quad (2)$$

Where P_{peak} represents peak output power, $P_{average}$ means average output power. $E[\cdot]$ denotes the statistical expected value, $\max[\cdot]$ gives the highest value among the samples, x_n represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols X_k . Mathematical, x_n is expressed as:

$$x_n = \frac{1}{\sqrt{n}} \sum_{k=0}^{N-1} X_k W_N^{nk} \quad (3)$$

For an OFDM system with N sub-carriers, the peak power of received signals is N times the average power when phase values are same. This requires that system devices, such as power amplifiers, A/D converters and D/A converters, must have large linear dynamic ranges. If this is not satisfied, a series of undesirable interference is encountered when the peak signal goes into the non-linear region of devices at the transmitter, such as high out of band radiation and inter-modulation distortion. PAPR reduction techniques are therefore of great importance for OFDM systems. Also due to the large fluctuations in power output the HPA (high power amplifier) should have large dynamic range. The nonlinearity of HPA causes inter-carrier interferences (ICI) and thus out-of-band radiation. Accordingly, the BER performance is degraded.

A. Complementary cumulative distribution function

The Cumulative Distribution Function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of any PAPR technique. Normally, the Complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold $\Pr(PAPR > PAPR_0)$. By implementing the Central Limit Theorem for a multi-carrier signal with a large number of sub-carriers, the real and imaginary part of the time-domain signals have a mean of zero and a variance of 0.5 and follow a Gaussian distribution [11]. So Rayleigh distribution is followed for the amplitude of the multi-carrier signal, where as a central chi-square distribution with two degrees of freedom is followed for the power distribution of the system.

The CDF of the amplitude of a signal sample is given by

$$F(z) = 1 - \exp(-z) \quad (4)$$

The CCDF of the PAPR of the data block is desired is our case to compare outputs of various reduction techniques. This is given by

$$\begin{aligned} P(PAPR > z) &= 1 - P(PAPR \leq z) \\ &= 1 - F(z)^N \\ &= 1 - (1 - \exp(-z))^N \end{aligned} \quad (5)$$

III. PARTIAL TRANSMITS SEQUENCE (PTS)

In PTS approach, the input data block is partitioned into disjoint sub-blocks. The sub-carriers in each sub-block are weighted by phase rotations. The phase rotations are selected such that the PAPR is minimized. At the receiver, the original data are recovered by applying inverse phase rotations. In the PTS technique, an input data block of K symbols is partitioned into disjoint sub-blocks. The subcarriers in each sub-block are weighted by a phase factor for that sub-block. The phase factors are selected such that the PAPR of the combined signal is minimized. In order to implement this idea, the input data block of K symbols is partitioned into M pair wise disjoint blocks X_k , $k = 1, \dots, M$.

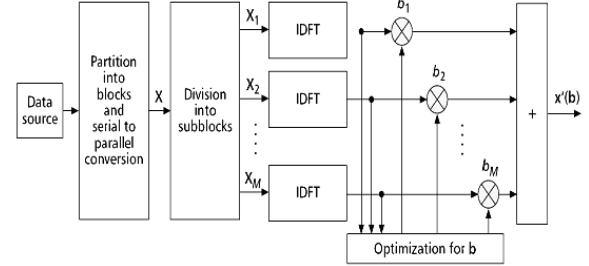


Figure 1: Block diagram of PTS technique.

Mainly, the total number of subcarriers included in any one of these sub-blocks X_k is arbitrary, but sub-blocks of equal size have been found to be an appropriate choice [13][14]. All subcarrier positions in X_k , which are already represented in another sub-block, are initialized to zero, so that $X = \sum_{k=1}^M X_k$.

Each sub-block is weighted by a set of rotation factors $b_k(u)$ where $u = 1, \dots, U$, so that a modified subcarrier vector $\hat{X} = \sum_{k=1}^M X_k b_k(u)$ is obtained, which represents the same information as X , if the set $b_k(u)$ is known for each u and k . The phase factors are selected such that the PAR of the combined signal is minimized (Fig-1).

Mathematically, it is expressed as:

$$\{b_1(u), b_2(u), \dots, b_M(u)\} = \operatorname{argmin}_u \left(\max_{0 \leq n < N_{u-1}} \left| \sum_{k=1}^M \text{IDFT}(X_k) b_k(u) \right| \right) \quad (4)$$

$$\text{Where, } b_u(u) = e^{j\phi(u_k)}, \quad \phi(u_k) \in (0, 2\pi)$$

Resulting in the optimum transmit sequence

$$\hat{X}(u_{opt}) = \sum_{k=1}^M \text{IDFT}(X_k) b_k(u) \quad (5)$$

Where u_{opt} is the phase vector that gives the greater reduction.

Hence, $U^{(M-1)}$ is the amount of sets of phase factors that are evaluated to find the best case. The total complexity increases exponentially with the number of sub-blocks M . The receiver needs to know the set $b_k(u)$. Hence, an unambiguous representation of it must be transmitted to the receiver. As a consequence, the amount of bits as side information is $\lceil \log_2 U^{(M-1)} \rceil$. Fig-1 represents the block diagram of PTS algorithm. From the left side of diagram, the data information in frequency domain X is separated into M non-overlapping sub-blocks and each sub-block vectors has the same size N . Hence, we know that for every sub-block, it contains N/M nonzero elements

and set the rest part to zero. The $argmin(.)$ is the judgment condition that output the minimum value of function. In this way we can find the best \mathbf{b} so as to optimize the PAPR performance. The additional cost we have to pay is the extra $M-1$ times IDFTs operation.

The optimization is achieved by searching thoroughly for the best phase factor. Theoretically, $\mathbf{b} = [b_1, b_2, \dots, b_M]$ is a set of discrete values, and numerous computation will be required for the system when this phase collection is very large. For example, if φM contains W possible values, theoretically, \mathbf{b} will have W^M different combinations, therefore, a total of $M \cdot W^M$ IFFTs will be introduced [11]. By increasing the M , W , the computational cost of PTS algorithm will increase exponentially.

For instance, define phase factor b_M contains only four possible values, that means $b_M \in \{\pm 1, \pm j\}$, then for each OFDM symbol, $2 \cdot M - 1$ bits are transmitted as side information. Therefore, in practical applications, computation burden can be reduced by limiting the value range of phase factor $\mathbf{b} = [b_1, b_2, \dots, b_M]$ to a proper level. At the same time, it can also be changed by different sub-block partition schemes [12]. There are three kinds of sub-block partitioning method schemes: Adjacent sub-block partitioning, Interleaved sub-block partitioning and Pseudo-random sub-block partitioning [13][14].

IV. SIMULATION AND DISCUSSION

To evaluate and compare the performance of PTS technique for different number of sub-blocks, we simulate it using MATLAB. PAPR reduction performance depends on the number of sub-blocks V and the number of possible phase value W [11]. In this paper we study the effects of sub-block numbers.

Table 1 Parameters used in PTS simulation

Parameters	Values used
Number of sub-carriers (N)	64, 128, 256, 512
Oversampling factor	4
Modulation scheme	QAM
Number of sub-blocks used	4, 8, 16, 32

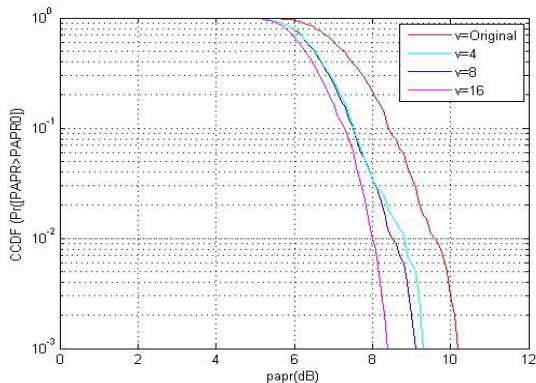


Figure 2: Comparison of PAPR reduction performance with different values of V (4, 8, 16) and Subcarrier $N=64$.

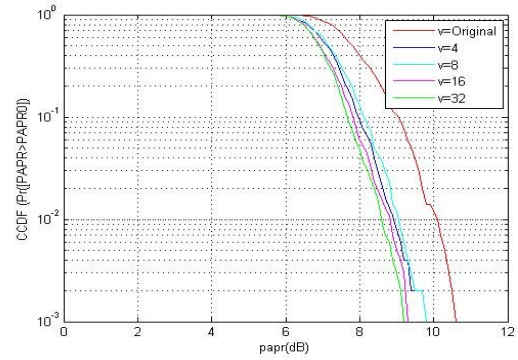


Figure 3: Comparison of PAPR reduction performances with different values of V (4, 8, 16, 32) and Subcarrier $N=128$

Fig. 2 shows the simulation curve of PTS technique for the number of subcarrier $N=64$ and the data is transmitted using different number of sub-blocking factor. V takes the value of 1 (without adopting PTS method), 4, 8 and 16. It is seen from fig. 2 that with increase of branch number V , PAPR of the signal gets smaller and smaller. It is observed that increasing V leads to the improvement of PAPR reduction performance. When we use sub-block factor value $V=4$ then the PAPR value of the signal is about 1dB smaller than the original signal. Under the same condition, when sub-block number $V=8$ then PAPR value is 1.2dB smaller than the original signal. Finally when sub-block $V=16$ then the PAPR value of the signal is about 2dB smaller than the unmodified original signal.

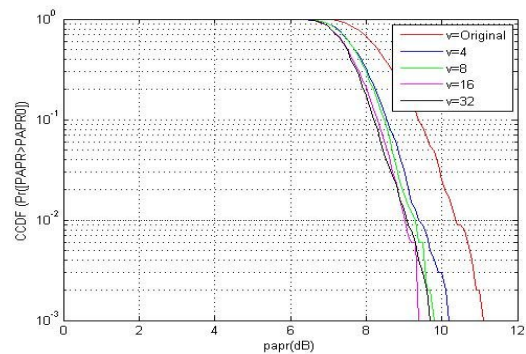


Figure 4: Comparison of PAPR reduction performance with different values of V (4, 8, 16, 32) and Subcarrier $N=256$.

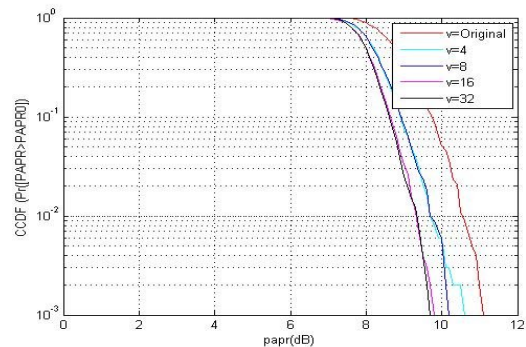


Figure 5: Comparison of PAPR reduction performances with different values of V (4, 8, 16, 32) and Subcarrier $N=512$.

Figure 3, 4 and 5 represents the PAPR reduction for four sub-blocking techniques. Figure 3 shows the sub-blocking effects of PTS technique when number of subcarrier N is 128. When $V=16$, the PAPR is reduce by an amounts 1.7dB from original unmodified signal. For sub-block numbers $V=32$, the PAPR value is approximately similar to $V=16$. Because as the sub-blocking factor V increases, it influences the complexity of the PTS algorithm. Similar effect is found for the subcarrier number $N=256$ (fig.-4). PAPR is reduced as the sub-block number is increased. When $V=16$, the PAPR is reduce by an amounts 1.6dB from original unmodified signal. For $V=32$ and $N=256$, PAPR is similar to $V=16$. As we further increases subcarriers $N=512$ (fig. 5), the PAPR effects reduced but less than for $N = 256$. From fig. 5, it is found that the PAPR is reduce about 1.5dB when $V=16$. When $V=32$ the PAPR value is approximately similar for the value of $V=16$.

Form our study it is obvious that as we increase the value of sub-blocking factor V , the complexity of the PTS algorithm is increase. The sub-blocking factor value $V \geq 16$, the PAPR reduction performance of OFDM signal using PTS algorithm is not be considerably improved.

V. CONCLUSION

There are several techniques to reduce the PAPR in OFDM transmission system. All PAPR reduction techniques have advantages and limitations. PTS technique can significantly improve the PAPR problem. Success of PTS technique depends on the number of sub-blocks and the number of possible phase values. In this paper we study PTS technique for different number of sub-blocks. The simulations show that PAPR reduced when the number of sub-blocks are increases. In PTS technique, when number of sub-blocks more than 16, PAPR of OFDM does not reduced significantly. The complexity increases exponentially with the number of sub-blocks. Moreover PTS technique performance gradually decreases when number of subcarriers of OFDM increases.

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