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FINITE DIFFERENCE ANALYSIS OF TRANSIENT FREE CONVECTIVE MHD HEAT TRANSFER FLOW THROUGH POROUS MEDIUM WITH HEAT GENERATION

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Abstract: The present paper is concerned with analytical solution of one-dimensional unsteady laminar boundary layer MHD flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate in presence of transverse magnetic field with heat source through porous medium. The vertical plate and the medium of flow are considered to be porous. The fluid is assumed to be optically thin and the magnetic Reynolds number is considered small enough to neglect the induced hydro magnetic effects. The governing boundary layer equations are first converted to dimensionless form and then solved by finite difference technique. The solution for transient velocity, temperature, skin friction and Nusselt number are illustrated and are presented in graphs for various sets of physical parametric values.

Keywords: magnetic Reynolds, transient velocity, skin friction, Porous Medium

I. INTRODUCTION

The deep interest in the porous medium is easily understandable since porous medium is used in vast applications, which covers many engineering disciplines. For instance, applications of the porous media includes, thermal insulations of buildings, heat exchangers, solar energy collectors, geophysical applications, solidification of alloys, nuclear waste disposals, drying processes, chemical reactors, energy recovery of petroleum resources, etc. In view of this some of the authors considered Chaudhary and Jain [1] has MHD heat and mass diffusion flow by natural convection past a surface embedded in a porous medium, Rajesh et.al [2] examined transient MHD free convection flow and heat transfer of nano-fluid past an impulsively started vertical porous plate in the presence of viscous dissipation, Ibrahim [3] studied effects of chemical reaction on dissipative radiative MHD flow through a porous medium over a non-isothermal stretching sheet, Ch Kesavaiah et. al. [4] analyzed effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction, Ch Kesavaiah et. al. [5] observed effects of radiation and free convection currents on unsteady Couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium.

The study of heat generation effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reaction. Bhavana et. al. [6] determined the Soret effect on

free convective unsteady MHD flow over a vertical plate with heat source, very recently Chenna Kesavaiah and Jahagirdar [7] studied radiation absorption and chemical reaction effects on MHD flow through porous medium past an exponentially accelerated inclined plate. Srinathuni Lavanya and Chenna Kesavaiah [8] Magnetic field and Radiation effects on MHD Free convection heat and mass transfer flow through a porous medium with chemical reaction, Srinathuni Lavanya and Chenna Kesavaiah [9] Heat transfer to MHD free convection flow of a visco-elastic dusty gas through a porous medium with chemical reaction, recently Ashish Paul [10] studied transient free convective MHD flow past an exponentially accelerated vertical porous plate with variable temperature through a porous medium, Whitehead [11] Observations of rapid means flow produced mercury by a moving heater, Sulochana et. al. [12] Heat and mass transfer flow of a viscous fluid through a porous medium in a wavy channel with traveling thermal waves.
Present work is on analytical solution of one-dimensional unsteady laminar boundary layer MHD flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate in presence of transverse magnetic field and heat source through porous medium.

II. FORMULATION OF THE PROBLEM

An unsteady one – dimensional laminar free convection flow of a viscous incompressible fluid past an infinite vertical porous plate through a porous medium with variable temperature and heat source is considered. The $x-$axis is being taken vertically upwards along the vertical plate and $y-$axis to be normal to the plate. The physical model and coordinate system of the flow problem is shown in figure (1).

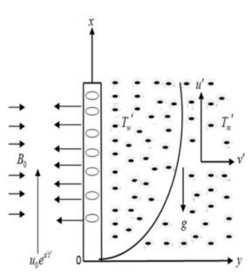

Figure (1): Physical model and coordinate system

Initially, it is assumed that the plate and fluid are at the same temperature T_∞ in the stationary condition. At $t' \geq 0$, the plate is exponentially accelerated with a velocity $u' = u_0{}' \exp(a't')$ in its own plane and the plate temperature is raised linearly with time t. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, so that the magmatic Reynolds number is much less than unity and hence the induced magnetic field is negligible in comparison with the applied magnetic field. The fluid considered here is a gray, absorbing/emitting radiation but a non scattering medium. The viscous dissipation is also assumed to be negligible in the energy equation as the motion is due to free convection only. It is also assumed that all the fluid properties are constant except for the density in the buoyancy term, which is given by the usual Boussinesq's approximation. Under these assumptions the governing boundary layer equations are

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = v' \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) - \frac{\sigma B_0^2}{\rho} u' - \nu \frac{u'}{k'} \qquad (1)$$

$$\rho C_p \left(\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = \kappa \frac{\partial^2 T'}{\partial y'^2} - Q_0 \left(T' - T'_\infty\right) \qquad (2)$$

With the following initial and boundary conditions:

$$\begin{aligned} &u' = 0, \quad T' = T'_\infty && \forall \quad y', \quad t' \leq 0 \\ &u' = u_0 \exp\left(a't'\right), T' = T'_\infty\left(T'_w - T'_\infty\right) At' && at \;\; y' = 0 \\ &u \to 0, \quad T \to T'_\infty && as \;\; y \to \infty \end{aligned} \qquad (3)$$

Where, $A = \dfrac{u_0^1}{\nu}$

In order to write the governing equations, initial and the boundary conditions the following non-dimensional quantities are introduced.

$$\begin{aligned} &Y = \frac{y' v_o}{\nu},\; U = \frac{u'}{u_w},\; t = \frac{t' u_0}{\nu},\; \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\; Gr = \frac{g\beta\nu\left(T_w{}' - T'_\infty\right)}{u_0^3} \\ &M = \frac{\sigma B_0^2 \nu}{\rho}, k = \frac{k' u_0^2}{\nu^2}, R = \frac{16 a^* \sigma T'^3_\infty}{\kappa u_0^2}, a = \frac{a'\nu}{u_0^2}, \gamma = -\frac{\nu'}{u_0}, \Pr = \frac{\mu C_p}{k} \end{aligned} \qquad (4)$$

In view of (4) the equations (1) and (2) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial t} - \gamma \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + Gr\,T - M\,U - \frac{1}{k} U \qquad (5)$$

$$\frac{\partial \theta}{\partial t} - \gamma \frac{\partial \theta}{\partial Y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} - Q\theta \qquad (6)$$

With following initial and boundary conditions:

$$\begin{aligned} &U = 0, \quad T = 0 && \forall \quad Y, \quad t' \leq 0 \\ &U = \exp\left(at\right), \theta = t && t > 0, at \; Y = 0 \\ &U \to 0, \quad \theta \to 0 && as \;\; Y \to \infty \end{aligned} \qquad (7)$$

where Gr is the thermal Grashof number, $\Pr$ is the fluid Prandtl number, Q is the heat source parameter, M is the magnetic parameter, a' is the accelerating parameter, a dimensionless accelerating parameter, a^* absorption coefficient, c_p specific heat at constant pressure, B_0 transverse magnetic field strength, g acceleration due to gravity, κ thermal conductivity of the fluid, k' permeability parameter, k dimensionless permeability parameter, t' time, t dimensionless time, T' temperature, T dimensionless temperature, T'_w is the temperature of the plate, T'_∞ is the temperature of the fluid far away from the plate, u' is the $x-$component of the velocity, u'_0 velocity of the plate, U is the dimensionless velocity, V' is the $y-$component of velocity, y' is the coordinate axis normal to the plate, Y is the dimensionless coordinate axis normal to the plate, β is the volumetric coefficient of thermal expansion, γ is the suction parameter, ν is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity of fluid.

III. METHOD OF SOLUTION

Equations (5) – (6) are coupled, non – linear partial differential equations and these cannot be solved in closed – form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved by finite difference method.

Let us consider a rectangular region with y varying from 0 to $y_{\max}(=40)$, where $y_{\max}$ represents to $y=\infty$. The region to be examined in (y,t) space is covered by a rectangular grid with sides parallel to axes with Δy and Δt, the mesh sizes along $y-$direction and time $t-$direction, respectively. The equivalent finite difference schemes of equations for (5) – (6) are as follows:

$$\left(\frac{U_{i,j+1}-U_{i,j}}{\Delta t}\right)-\gamma\left(\frac{U_{i,j+1}-U_{i,j}}{\Delta y}\right)=\left(\frac{U_{i-1,j}-2U_{i,j}+U_{i+1,j}}{(\Delta y)^2}\right)+Gr(\theta_{i,j}) \qquad (8)$$
$$-M(U_{i,j})-\frac{1}{k}(U_{i,j})$$

$$\left(\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}\right)-\gamma\left(\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta y}\right)=\frac{1}{\Pr}\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta y)^2}\right)-Q(\theta_{i,j}) \qquad (9)$$

Index i refer to y and j refers to time; the mesh system is divided by taking $\Delta y = 0.1$

From the initial conditions in (7) we have the following equivalent

$$\begin{aligned} &u(i,0)=e^{at},\theta(i,0)=t && \textit{for all } i \\ &u(0,j)=0,\theta(0,j)=0 && \textit{for all } j \\ &u(i_{\max},j)=0,\theta(i_{\max},j)=0 && \textit{for all } i \end{aligned} \qquad (10)$$

(here $i_{\max}$ was taken as 200), the velocity at the end of time step viz, $u(i,j+1)(i=1,200)$ is coupled form (8) in terms of velocity and temperature at points on the earlier time – step. After that $\theta(i,j+1)$ is computed for (9); the procedure is repeated until $t=0.5(i.e.\ j=500)$, during the computation Δt was chosen as 0.001.

IV. RESULTS AND DISCUSSION:

In order to get an insight into the physical solution of the problem, the numerical computation of velocity profiles, temperature profile, skin friction and Nusselt number are obtained for different values of magnetic field parameter (M), Grashof number (Gr), accelerating parameter (a), suction parameter (γ), permeability parameter (K), heat source parameter (Q) and time (t) are presented graphically in figures (2) – (13). The transient velocity profiles for different values of Grashof number (Gr) are shown in figure (2). The Grashof number signifies the relative effect of the buoyancy force to the hydrodynamic viscous force. The positive values of Grashof number correspond to cooling of the plate and the negative values of Grashof number correspond to heating of the plate by free convection. As expected, it is found that an increase in the Grashof number lead to increase in the velocity due to enhancement in the buoyancy force. The transient velocity profiles for different values of magnetic parameter (M) are depicted in figure (3). It is observed form this figure that an increase in magnetic field leads to decrease in the velocity profiles for both the cases of cooling $(Gr=2)$ and heating $(Gr=-2)$ of the porous plate. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. The effects of suction parameter on velocity profiles illustrated in figure (4), it is found here that velocity decreases with increase of the suction parameter for both cases of cooling $(Gr=2)$ and heating $(Gr=-2)$ of the porous plate. The effect of permeability parameter (K) and time (t) on velocity profiles are depicted in figure (5). It can be seen that the velocity increases with increase of permeability parameter and time. The transient velocity profiles for different values of accelerating parameter (a) and heat source parameter (Q) are plotted in figure (6). It is determined that the velocity decreases with increasing in radiation parameter but increasing with increasing in accelerating parameter. Figure (7) displays the velocity profiles for different values of Prandtl number $(\Pr)$, it is clear that the velocity decreases with increasing values of Prandtl number. Effects of radiation parameter (R), suction parameter (a), Prandtl number (Pr) and time (t) on temperature profiles are shown in figure (8), (9), (10) and (11) respectively. It is observed form these figures that temperature decrease with increased values of radiation parameter, suction parameter and Prandtl number, but increases with increased values of time. Effect of heat source parameter on skin friction is presented in figure (12). It is observed form this figure that skin friction decreases with increase of heat source parameter in case of cooling of the porous plate. The Nusselt number for different values of suction parameter is shown in figure (13). The rate of heat transfer decreases with increase of suction parameter.

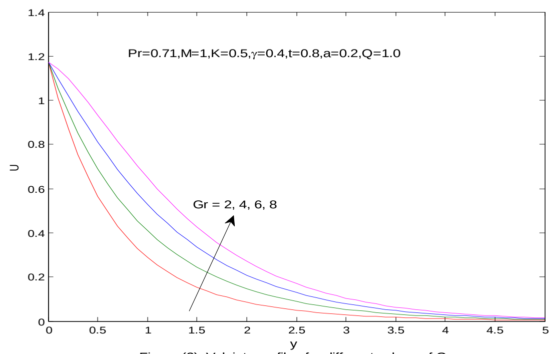

Figure (2): Veloicty profiles for different values of Gr

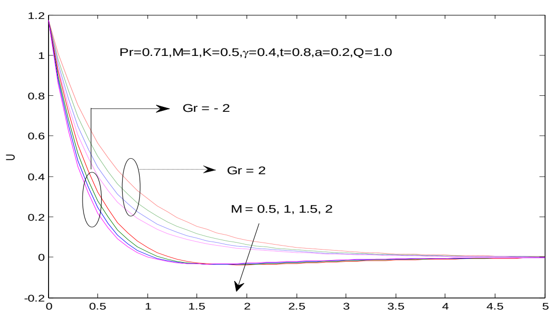

Figure (3): Veloicty profiles for different values of M

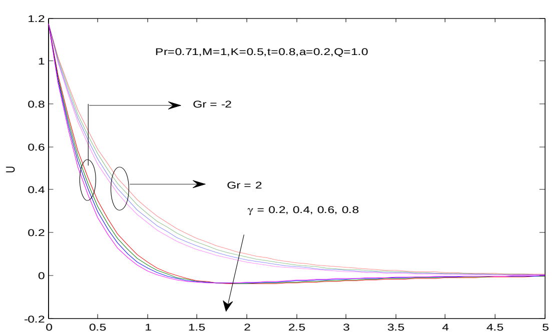

Figure (4): Veloicty profiles for different values of γ

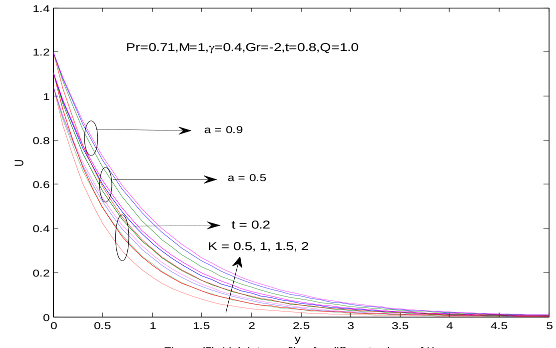

Figure (5): Veloicty profiles for different values of K

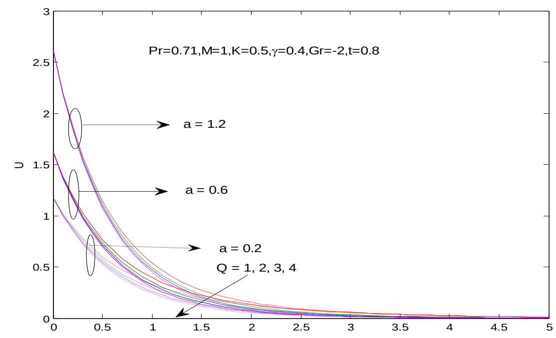

Figure (6): Veloicty profiles for different values of Q

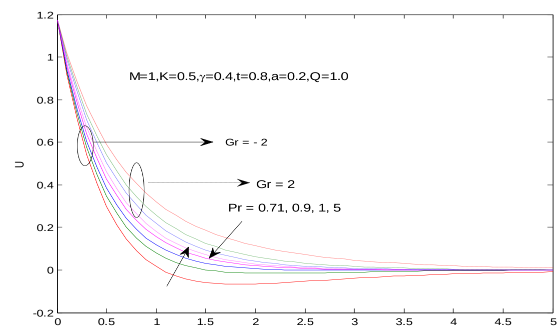

Figure (7): Veloicty profiles for different values of Pr

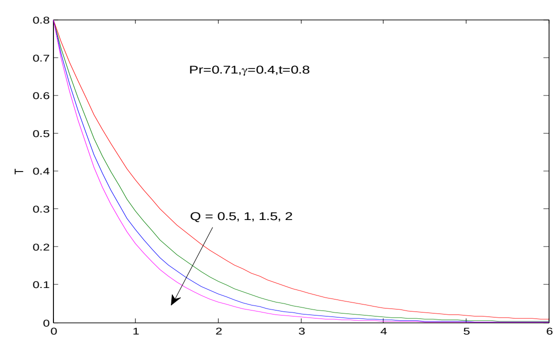

Figure (8): Temperature profiles for different values of Q

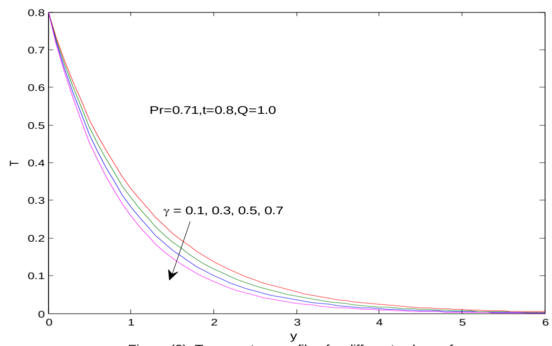

Figure (9): Temperature profiles for different values of γ

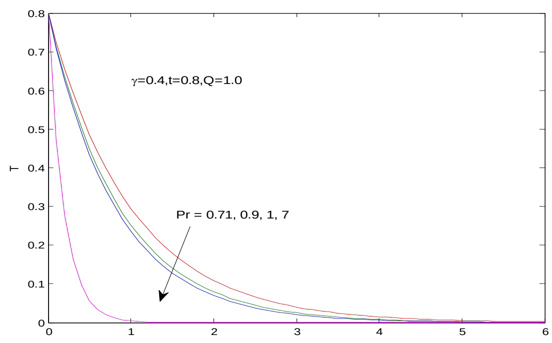

Figure (10): Temperature profiles for different values of Pr

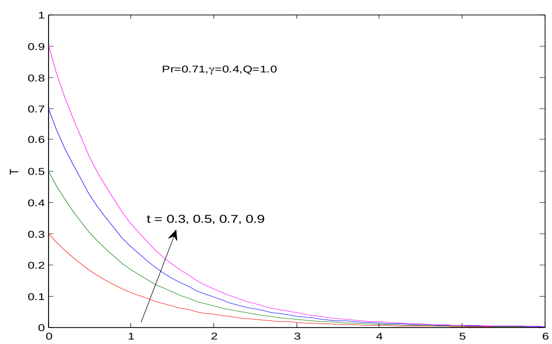

Figure (11): Temperature profiles for different values of t

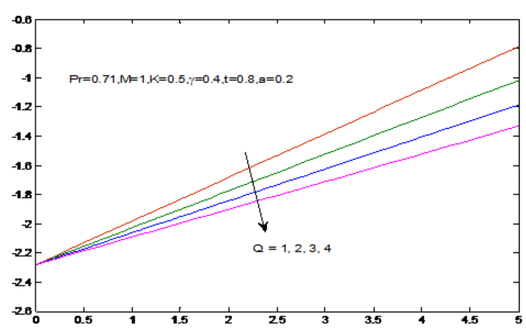

Figure (12): Skin Friction for different values of Q

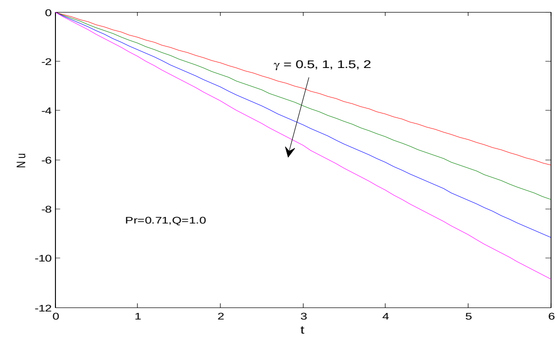

Figure (13): Nusselt number for different values of γ

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