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## ASSESSING WORST WEATHER BY ESTIMATING VALUE-AT-RISK USING HETEROSCEDASTIC PROCESS

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**Abstract:** A substantial issue in modern risk management is the measurement of risks. Specify, the requirement to quantify risk discovers in many different contexts. For instance, a regulator measures the risk exposure of a government institution in order determining the maximum value from any phenomenon occurred as a tool against unexpected losses. Particularly attention will be given to Value-at-Risk (V@R). Mostly, implementation of V@R is in financial cases, as potential alarm of institution to anticipate the magnitude of risk. Combining V@R with the forecast function of AR-ARCH processes, this paper proposes a new implementation of estimative-V@R and improved-V@R to compute heavy rain as representation of worst weather, which has the same future goal providing funds to anticipate financial losses. There are limited researches related to heavy rain forecast based on constructing a process by considering risk of with modifying some mathematics equations. We consider an overview of the existing approaches to measure V@R of weather data involving time series process and some stochastic expansion. We present V@R using AR and heteroscedastic processes ARCH considering the changes of data volatility. We consider an estimative prediction limit to determine an improved prediction limit with better conditional coverage properties. The parameter estimator of AR-ARCH is assumed to have the same asymptotic distribution as the conditional maximum likelihood estimator. This paper deals with calculation coverage probability to validate  $\alpha$ -V@R performance.

**Keywords:** AR, ARCH, coverage probability, forecasting, Value-at-Risk.

### I. INTRODUCTION

When dealing with random variables, one of the characteristics that has to be considered is the maximum value observed. In many cases the maximum value is related to a loss or a risky condition. A risk measurement becomes an important study in data analysis, since it is often associated with investment and is not uncommonly associated with public (society) funding. A general viewpoint of risk is a risk mostly proportional to profit, the truly greatest risk happens when the company or a government or even individual does not dare to take risks.

And risk management will never be effective if it is not accompanied by the ability to measure a risk itself. One of measuring tools that used to quantify the risk condition is Value-at-Risk (V@R). This is very clear that V@R represents as a maximum value can be tolerated at some confidence levels ( $\alpha$ ).

Independently of any context, risk relates strongly to uncertainty, and hence to the notion of randomness. As the observation data is used a rainfall data from Bandung regency to calculate prediction and measure the performance of V@R. In 2017, there are many issues of disasters occurred in Indonesia related to the bad weather,

some of the concerns during 2017-2018 are flood and landslide disasters. Natural disasters such as floods are one of event related to risk due to high levels of unanticipated heavy rain. It should be clear from the outset that good risk measurement is a must.

Suppose  $Y = Y_1, Y_2, \dots, Y_n, n \in N^+$  the observable random rainfall vector, the future Value-at-Risk (V@R) of rainfall follows a cumulative distribution  $F_y$ , depending on the unknown parameter  $\omega$ . An  $\alpha$ -prediction interval for V@R (henceforth quantile V@R), in particularly, an  $\alpha$ -prediction limit  $q_{n+1}^\alpha$ , exactly or approximately,

$$P_\omega(Y_{n+1} \leq q_{n+1}^\alpha(\omega|Y_1, Y_2, \dots, Y_n)) = \alpha$$

Some researchers concern to introduce volatility-time series processes and stochastics expansions using Taylor and Maclaurin series as a part of the process of managing risk [1].

Researchers and practitioners have started to forecast weather using Soft Computing and Data mining approach: see, for example the previous research point out that a comparative study of grammatical evolution and adaptive neuro-fuzzy inferences system for forecasting rainfall in Bandung [2]. Then, some researcher implemented a local regression smoothing and Fuzzy-grammatical evolution for rainfall forecasting [3]. There are limited researches related to rainfall forecast based on constructing a time series models by considering risks (heavy rain) forecast with modifying some mathematics equations.

Banrdoff *et al.* interpret the upper prediction limit approach that achieve third – order accuracy [4]. Related to Huang *et al.*, V@R is managed by using heteroscedastic time series GARCH process and Copula approach. To determine the performance of V@R – based on heteroscedastic process is calculated by the coverage probability, as noted by Vidoni [5-6]. The previous research considers an alternative measure using Expected Shortfall over V@R, due to its capability to handle magnitude risk under the result of coverage probability [7]. As noted that rainfall observation follows time series observation. According to McNeil *et al.*, ARCH is a time series process that can accommodate for changing volatility behavior on data observation [8]. A volatility describes the amount of data change, in this case of rainfall data, expressed by a conditional standard deviation.

Therefore, in this paper we consider time series processes AR-ARCH with stochastic expansion for calculating heavy rainfall V@R-factor change data. We begin by looking more systematically at the description properties of V@R in Section II. In Section III, we review essential concept of volatility in the analysis of time series, such as AR-ARCH processes. We then devote section IV to simulate modified V@R based on AR-ARCH processes including a summary of the V@R-coverage probability.

## II. DESCRIPTION OF V@R-BASED MODIFIED

### UPPER PREDICTION LIMIT

Suppose the observable random rainfall vector  $Y = (Y_1, Y_2, \dots, Y_n), n \in N^+$ , the future Value-at-Risk (V@R),  $q_{n+1}^\alpha(\omega|Y_1, Y_2, \dots, Y_n)$ , follows a cumulative distribution  $F_y$  depending on the unknown parameter  $\omega$ . Firstly, our aim is to find an upper limit  $q_{n+1}^\alpha(\omega|Y_1, Y_2, \dots, Y_n)$ , for  $Y_{n+1}$ , such that it has coverage probability equal to  $\alpha$  and refers to the joint distribution of  $(Y^n, Y^{n+1})$ . Based on Eq. 1, we consider an  $\alpha$  as a coverage probability of the magnitude of heavy rainfall equal and or less than V@R. In case to anticipate a heavy rainfall, we examine calculating V@R,  $q_{n+1}^\alpha$ .

$$P_\omega(Y_{n+1} \leq q_{n+1}^\alpha(\omega|Y_1, Y_2, \dots, Y_n)) = \alpha \tag{1}$$

where  $\alpha \in (0,1)$  is fixed for all  $\omega$ . While  $\omega$  is unknown, we consider for estimate solutions and the simplest approach requires the estimative predictive density  $f(q; \omega|y)$ , an estimator of the right conditional density obtained by replacing  $\omega$  with an asymptotically efficient estimator  $\hat{\omega}$ . Thus, we noted the argument (due to Barndorff *et al.*) presenting that the coverage probability of  $q_{n+1}^\alpha$  that is  $\alpha + O(n^{-1})$  [4]. It is well known that unconditional and the  $\alpha$ -conditional coverage probability of  $\hat{q}_{n+1}^\alpha$  differ from  $\alpha$  by a term usulaly of order  $O(n^{-1})$  and prediction statements may be rather inaccurate for small. This section deals with calculation coverage probability of the estimative prediction limit  $\hat{q}_{n+1}^\alpha$ , it is possible to rewrite Eq. 1 as,

$$P_\omega(Y_{n+1} \leq q_{n+1}^\alpha(\omega|Y_1, Y_2, \dots, Y_n)) = E_\omega(F(q_{n+1}^\alpha(\omega|Y_1, Y_2, \dots, Y_n)))$$

With the above formulations, given an estimation statistic expressed,

$$G_\alpha(\hat{\omega}) = F(q_{n+1}^\alpha(\hat{\omega}|Y_1, Y_2, \dots, Y_n)) \tag{2}$$

where  $E_\omega$  is carried out, numerically, using the parametric bootstrap. Now, by applying a Taylor expansion for Eq. 2, considered as function of  $\hat{\omega}$ , may be shown that,

$$G_\alpha(\hat{\omega}) = G_\alpha(\omega) + \left. \frac{\partial G_\alpha(\hat{\omega})}{\partial \hat{\omega}_i} \right|_{\hat{\omega}=\omega} (\hat{\omega}_i - \omega_i) + \left. \frac{\partial^2 G_\alpha(\hat{\omega})}{\partial \hat{\omega}_p \partial \hat{\omega}_q} \right|_{\hat{\omega}=\omega} (\hat{\omega}_p - \omega_p)(\hat{\omega}_q - \omega_q) + \dots$$

where  $\omega_i$  denotes the-*i*-th component of parameter vector  $\omega$ , around  $\hat{\omega} = \omega$  and the taking the mean with respect to probability density  $g(\hat{\omega}; \omega, \alpha)$ , we establish an approximation for the  $\alpha$ -conditional coverage probability of  $\hat{q}_{n+1}^\alpha$

$$P_\omega(Y_{n+1} \leq q_{n+1}^\alpha(\hat{\omega}|Y_1, Y_2, \dots, Y_n)) = \alpha + \left. \frac{\partial G_\alpha(\hat{\omega})}{\partial \hat{\omega}_i} \right|_{\hat{\omega}=\omega} E(\hat{\omega}_i - \omega_i) + \left. \frac{1}{2} \frac{\partial^2 G_\alpha(\hat{\omega})}{\partial \hat{\omega}_p \partial \hat{\omega}_q} \right|_{\hat{\omega}=\omega}$$

$$E((\hat{\omega}_p - \omega_p)(\hat{\omega}_q - \omega_q)) + \dots$$

since exactly  $G_\alpha(\omega) = \alpha$  and we have  $d_\alpha(\omega)n^{-1} + \dots$  as correction factor of  $\alpha$ . Thus,

$$P_\omega(Y_{n+1} \leq q_{n+1}^\alpha(\omega|Y_1, Y_2, \dots, Y_n)) = \alpha + d_\alpha(\omega)n^{-1} + \dots \quad (3)$$

The  $O(n^{-1})$  term  $d_\alpha(\omega)n^{-1}$  is noted by Barndorff *et al.* and Vidoni, they reported two equivalent modifications [4][6],

$$E_\omega(\widehat{\omega} - \omega|Y_1, Y_2, \dots, Y_n) = b(\omega)n^{-1} + \dots$$

$$E_\omega((\widehat{\omega} - \omega)(\widehat{\omega} - \omega)^T|Y_1, Y_2, \dots, Y_n) = i^{-1}(\omega) + \dots$$

Then,  $d_\alpha(\omega)n^{-1}$  leads to

$$\frac{\partial G_\alpha(\widehat{\omega})}{\partial \widehat{\omega}_i} \Big|_{\widehat{\omega}=\omega} b(\omega)n^{-1} + \frac{1}{2} \frac{\partial^2 G_\alpha(\widehat{\omega})}{\partial \widehat{\omega}_p \partial \widehat{\omega}_q} \Big|_{\widehat{\omega}=\omega} i^{pq} + \dots$$

Some improvement for obtaining prediction limits with better asymptotic coverage properties have been shown by Kabaila-Syuhada [9],

$$q_{\alpha,n+1}^+(\omega|Y_1, Y_2, \dots, Y_n) = q_{n+1}^\alpha(\omega) - c_\alpha(\omega)n^{-1} \quad (4)$$

It is possible to obtain an approximate correct to order  $O(n^{-\frac{3}{2}})$  expressed as modification of the estimative prediction limit of Eq. 1,

$$c_\alpha(\omega) = \frac{d_\alpha(\omega)}{f(q_{n+1}^\alpha(\omega))}$$

The prediction limit Eq. 4 is defined to keep the additional  $O(n^{-1})$  term in Eq. 3, so that both the unconditional and the conditional coverage probability, is  $\alpha + O(n^{-\frac{3}{2}})$ . A bias  $b(\omega)n^{-1}$  of estimator  $\widehat{\omega}$  is surely affecting on calculation the correction  $c_\alpha(\omega)$ . Thus, the  $\alpha$ -quantile to third-order accuracy is

$$P_\omega(Y_{n+1} \leq q_{\alpha,n+1}^+(\omega|Y_1, Y_2, \dots, Y_n)) = \alpha + c_\alpha(\omega)n^{-1} + O(n^{-\frac{3}{2}}) + \dots$$

We recap the argument (due to Ueki-Fueda) showing the improvement prediction limit  $q_{\alpha,n+1}^+$  may be calculated as [10],

$$q_{n+1}^\alpha + \frac{\partial q_\alpha(\widehat{\omega})}{\partial \widehat{\omega}_i} \Big|_{\widehat{\omega}=\omega} (\widehat{\omega}_i - \omega_i) + \frac{1}{2} \frac{\partial^2 q_\alpha(\widehat{\omega})}{\partial \widehat{\omega}_p \partial \widehat{\omega}_q} \Big|_{\widehat{\omega}=\omega} (\widehat{\omega}_p - \omega_p)(\widehat{\omega}_q - \omega_q) + c_\alpha(\omega) + \dots$$

### III. COMPUTATION OF V@R COVERAGE-BASED CORRECTED FOR AR(1) PROCESS

We first consider  $\{Y_t\}_{t \geq 1}$  is the observed time series following Autoregressive (AR) process. The AR(1) process represents the combination of present and past observation at  $l$ -th order and satisfies,

$$Y_t = bY_{t-1} + \epsilon_t$$

The  $\epsilon_t$  are ‘innovation’ term that independent and identically distribution. Suppose that  $\{Y^{(t)}\}$  is a Markov process, where the observable data are  $Y_1, Y_2, \dots, Y_t$ . Notice the implication of variance AR(1) process that  $|b| < 1$ . Fix  $\alpha \in (0,1)$ , we can calculate  $\alpha$ -estimative V@R for  $Y_{t+1}$ , refers to Eq.2,

$$P_b(Y_{t+1} \leq q_{t+1}^\alpha(b|Y^{(t)})) = P_b(\epsilon_{t+1} \leq q_{t+1}^\alpha(b|Y^{(t)}) - \widehat{b}y_t)$$

$$\alpha = \Phi_\epsilon(q_{t+1}^\alpha(b|Y^{(t)}) - \widehat{b}y_t)$$

Using  $\Phi^{-1}$ , the  $\alpha$ -estimative V@R for  $Y_{t+1}$  satisfies,

$$q_{t+1}^\alpha(\widehat{b}|Y^{(t)}) = -\widehat{b}y_t + \Phi_\epsilon^{-1}(\alpha) \widehat{\sigma} \quad (5)$$

Since,  $\Phi$  denotes the non-negative distribution function and  $\widehat{b}$  is obtained by implementing conditional maximum likelihood, the result in Table 1. Then, we calculate  $\widehat{\sigma}$  as,

$$\frac{\sum_{t=2}^n (y_t + \widehat{b}y_{t-1})^2}{n-1}$$

TABLE I: PARAMETERS ESTIMATION OF AR AND ARCH

| Process | Parameters         | Value             |
|---------|--------------------|-------------------|
| AR(1)   | $\widehat{b}$      | 0.6392            |
|         | $\widehat{\sigma}$ | $2.3 \times 10^4$ |
| ARCH(1) | $\widehat{a}_0$    | $3.9 \times 10^4$ |
|         | $\widehat{a}_1$    | 0.3348            |

Now, we calculate the coverage probability of estimative V@R for AR(1) process respectively in Eq. 2 and Eq. 5,

$$P_b(Y_{t+1} \leq q_{t+1}^\alpha(\widehat{b}|Y^{(t)})) = P_b(\epsilon_{t+1} \leq (b - \widehat{b}) \frac{y_t}{\sigma} + \Phi_\epsilon^{-1}(\alpha) \frac{\widehat{\sigma}}{\sigma} \Big| Y^{(t)}) = y^{(t)}) = E_b \left( \Phi_\epsilon \left( (b - \widehat{b}) \frac{y_t}{\sigma} + \Phi_\epsilon^{-1}(\alpha) \frac{\widehat{\sigma}}{\sigma} \Big| Y^{(t)} = y^{(t)} \right) \right) \quad (6)$$

Then, the  $\alpha$ -improved V@R following Eq. 4 for AR(1) process may be calculated as,

$$q_{\alpha,t+1}^+(\widehat{b}|Y^{(t)}) = q_{t+1}^\alpha - c_\alpha(b)$$

We follow Kabaila-Syuhada to compute improved V@R for Autoregressive processes reducing the coverage error to  $O(n^{-\frac{3}{2}})$  [11],

$$P_b(Y_{t+1} \leq q_{\alpha,t+1}^+(\widehat{b}|Y^{(t)})) = P_b(\epsilon_{t+1} \leq b \frac{y_t}{\sigma} + \frac{q_{t+1}^\alpha(\widehat{b}|Y^{(t)})}{\sigma} \Big| Y^{(t)}) = y^{(t)}) = E_b \left( \Phi_\epsilon \left( b \frac{y_t}{\sigma} + \frac{q_{t+1}^\alpha(\widehat{b}|Y^{(t)})}{\sigma} \Big| Y^{(t)} = y^{(t)} \right) \right) \quad (7)$$

Fig. 1 shows the autocorrelation among data at lag 1-100, which is *cut off* in lag 2, then we consider to put orde-1 in time series processes, AR(1). It refers to the present

observation only depends on the past ( $t - 1$ ) observation. From Fig. 2, the V@R with 99% level of confidence is located above the rainfall data and describe the expectation maximum level of rainfall, with 108 limited observable rainfall. In this Fig. 2 that trends estimative V@R-AR not reflecting the maximum value with time-varying.

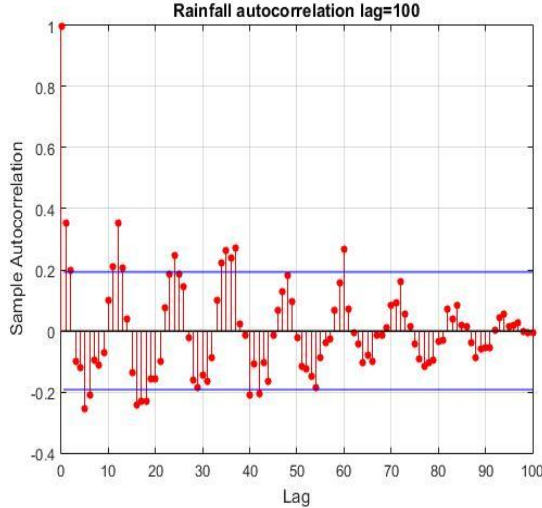


Figure 1: Plot rainfall autocorrelation (ACF) until lag 100

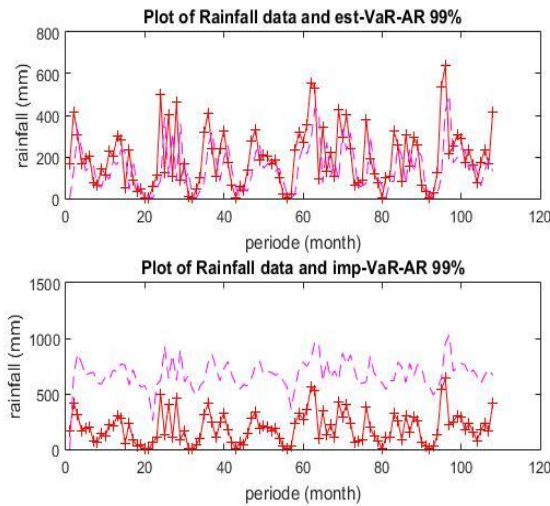


Figure 2: Estimated and improved V@R using AR(1) process

Thus, we can see improved V@R-AR approaching the extreme (maximum) value, but not precisely. The V@R performance present in Table 2, the coverage probability V@R-AR is quite different with level of confidence that given ( $\alpha$ ).

#### IV. COMPUTATION V@R COVERAGE-BASED CORRECTED FOR ARCH(1) PROCESS

We devote this section to univariate ARCH (*Autoregressive Conditional Heteroscedastic*) process for capturing the phenomenon of changing volatility. Volatility is often

formally called as the conditional standard deviation of financial returns given historical information [8]. Suppose the observable random rainfall vector  $Y = Y_1, Y_2, \dots, Y_t$  follow ARCH(1), may be written as,

$$Y_t = \sigma_t \epsilon_t$$

$A\sigma_t$  well known as volatility and  $\epsilon_t$  be *strict white noise*. The process  $\{Y_t\}$  is an ARCH(1) process if it is strictly stationary and if it satisfies,

$$\sigma_t^2 = a_0 + a_1 Y_{t-1}^2 \quad (8)$$

Let some confidence level  $\alpha \in (0,1)$ , the estimative V@R-ARCH(1) of rainfall data, formally,

$$P_a \left( Y_{t+1} \leq q_{t+1}^\alpha(a|Y^{(t)}) \right) = P_a \left( \epsilon_{t+1} \leq \frac{q_{t+1}^\alpha(a|Y^{(t)})}{\sigma_{t+1}} \right)$$

$$\alpha = \Phi_\epsilon \left( \frac{q_{t+1}^\alpha(a|Y^{(t)})}{\sigma_{t+1}} \right)$$

Thus,

$$q_{t+1}^\alpha(\hat{a}|Y^{(t)}) = \sigma_{t+1} \Phi_\epsilon^{-1}(\alpha) = \sqrt{\hat{a}_0 + \hat{a}_1 y_t^2} \Phi_\epsilon^{-1}(\alpha) \quad (9)$$

where  $\Phi$  is non-negative cumulative distribution of  $\epsilon_t$ ,  $a_i, i = 0, 1$  denotes the parameters of ARCH(1) that carried out using conditional maximum likelihood in Table 1.

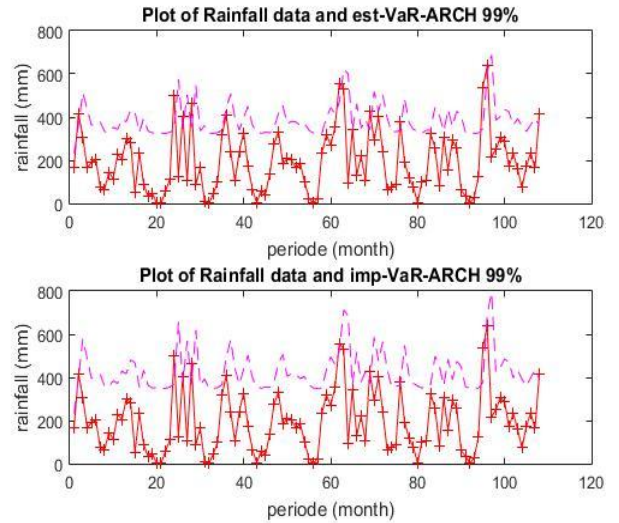


Figure3: Estimated and improved V@R using ARCH(1) process

Moreover, to calculate V@R-based on improved prediction limit, we have to discover  $c_\alpha(\omega)$  in Eq. 4, which may be expressed as,

$$\begin{aligned} P_a \left( Y_{t+1} \leq q_{a,t+1}^+(\hat{a}|Y^{(t)}) \right) &= P_a \left( \epsilon_{t+1} \leq \sqrt{\frac{\hat{a}_0 + \hat{a}_1 y_t^2}{a_0 + a_1 y_t^2}} \Phi_\epsilon^{-1}(\alpha) \middle| Y^{(t)} = y^{(t)} \right) \\ &= E_a \left( \Phi_\epsilon \left( \sqrt{\frac{\hat{a}_0 + \hat{a}_1 y_t^2}{a_0 + a_1 y_t^2}} \Phi_\epsilon^{-1}(\alpha) \middle| Y^{(t)} = y^{(t)} \right) \right) \end{aligned} \quad (10)$$

We simulate this conditional expectation using the method of Kabaila-Syuhada [5]. From Fig.3, the V@R data with a 99% confidence is surely higher than that with a 95% level

of confidence. We can see, comparing with V@R-AR(1), estimative and improved V@R-ARCH(1) describes the high level of rainfall data series quite well following the time-varying of data. Then, we compute coverage probability from estimative and improved V@R involving AR(1) and ARCH(1), based on Eq. 10. Then, we assess coverage probability as an  $\alpha$ -realization to measure the V@R performance. The following is the comparison a coverage probability between AR and ARCH based on a given level of confidence ( $\alpha$ ).

TABLE II: COVERAGE PROBABILITY BASED ON VARIOUS LEVELS OF CONFIDENCE

|         | $\alpha$ | $\hat{\alpha}$ -estimated | $\hat{\alpha}$ -improved |
|---------|----------|---------------------------|--------------------------|
| AR(1)   | 0.90     | 0.843                     | 0.997                    |
|         | 0.95     | 0.898                     | 0.997                    |
|         | 0.99     | 0.963                     | 0.991                    |
| ARCH(1) | 0.90     | 0.878                     | 0.910                    |
|         | 0.95     | 0.941                     | 0.951                    |
|         | 0.99     | 0.970                     | 0.992                    |

We show that in Table 2, implementation of coverage probability to rainfall data, both with an estimative an improved V@R, ARCH(1) better than AR(1) due its closeness of coverage probability ( $\hat{\alpha}$ ) to a given level of confidence ( $\alpha$ ).

### V. CONCLUSIONS

Value-at-risk (V@R) represents a methodology describing the maximum value of random variable and has become one of the most confidence tools to handle risk factors. We deal with an  $\alpha$  as the coverage probability of the magnitude of heavy rainfall equal and or less than V@R. An  $\alpha$  upper prediction limit for future value  $Y_{n+1}$  is such that exactly,

$$P_{\omega}(Y_{n+1} \leq q_{n+1}^{\alpha}(\omega|Y_1, Y_2, \dots, Y_n)) = \alpha$$

In case to anticipate a heavy rainfall, we examine calculating V@R,  $q_{n+1}^{\alpha}$ . Combining V@R with the forecast function of AR-ARCH processes, this paper proposes a new implementation of estimative-V@R and improved-V@R to estimate heavy rain representing worst weather (specified rainfall data in Indonesia). The empirical results show that, comparing between AR and ARCH processes, the ARCH process captures the V@R more successfully, considering coverage probability for each level of confidence. The coverage probability is discovered by calculating bias parameters estimation, using maximum likelihood and Taylor expansions (see Barndorff *et al.* and Vidoni) [4][6]. In addition, estimative and improved V@R-ARCH(1) describes the heavy rain of rainfall data series quite well.

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