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STABILITY ANALYSIS OF PALM OIL CONTROL MODEL WITH FELLING EFFECT

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Abstract: The stability analysis of logistic growth model is a norm in plantation and eco-biological environment. It can be ecologically stable and sustained in a long run. However, any additional variation to the model may fluctuate the economy over time. An optimal control felling rate model was successfully created for palm oil plantation based on the logistic growth model. The model can do well in representing the plantation but the stability is not yet analysed, mainly due to the presence of felling rate as the new added criterion. This study analysed the stability of equilibrium point of the optimal control felling rate model. The equilibrium points were identified and the small perturbation around the equilibrium points was geometrically computed. The analysis of small perturbation of felling rate showing equilibrium point decay exponentially indicates that the model was ecologically stable with the presence of control felling rate. The trajectories of palm oil biomass versus time verified the stability of equilibrium points with the presence of felling rate.

Keywords: Stability Analysis; Logistic Equation; Palm Oil Plantation; Ordinary Differential Equation; Optimal Control Model.

I. INTRODUCTION

The stability analysis of logistic growth model is a norm in plantation and eco-biological environment. The stability of population dynamics, equilibrium states and the stability of its function have been traditionally analysed using [1, 2, 3, 4]. Normally, the interests in population models are the equilibrium states and convergences towards it states. This paper emphasizes on the stability of palm oil model with the presence of felling effect. The model is developed based on the logistic growth model and the theory of optimal control model. Historically, following the original of population growth model of $\frac{dP}{dt} = aP$, the logistic growth model is invented by Malthus [5]. The P is denoted as the initial population while a represents the growth constant. Later on Verhulst [6] modified the model of Malthus by considering carrying capacity into the new model. This is to make a population size proportionate to a new term $\frac{a-bP(t)}{a}$, which does not only depend on the population size, but also distance of the size from the upper limit. To date, most successful predictive models are made based on extended forms of the classical Verhulst logistic growth equation. This logistic equation anticipates a limit on population growth as such

$$\frac{dP}{dt} = aP(1-\frac{P}{K}) \quad (1)$$

where $P_0 = P(0)$ at $t = 0$, $K = \frac{a}{b}$ is the carrying capacity and $b = \frac{a}{K}$ is a constant

$$\frac{dP}{dt} = aP - \frac{aP^2}{K}$$
$$= \frac{aKP - aP^2}{K}$$
$$= bKP - bP^2$$
$$= aP - bP^2$$
$$= P(a - bP)$$
$$\frac{dP}{P(a-bP)} = dt$$
$$\frac{1}{P(a-bP)}.dP = dt$$

to simplify $\frac{1}{P(a-bP)}$, the partial fractions is used as follows

$$\frac{1}{P(a-bP)} = \frac{A}{P} + \frac{B}{(a-bP)} \quad (2)$$

$$1 = A(a-bP) + BP$$
$$A(a-bP) + BP = 1$$
$$Aa - AbP + BP = 1$$

if $P = 0$, $Aa = 1$ and $A = \frac{1}{a}$, while
if $P = 1$, $B = \frac{b}{a}$
replace $A = \frac{1}{a}$ and $B = \frac{b}{a}$ into the equation 2

$$\frac{1}{P(a-bP)} = \frac{1}{aP} + \frac{b}{a(a-bP)}$$
$$\int dt = \int \frac{1}{aP} dP + \int \frac{b}{a(a-bP)} dP$$
$$\int dt = \frac{1}{a}\int \frac{1}{P} dP + \frac{1}{a}\int \frac{b}{(a-bP)} dP$$

let $u = a - bP$, $\frac{du}{dP} = -b$ and $du = -bdP$ thus

$$\frac{1}{a}\int \frac{1}{P} dP + \frac{1}{a}\int \frac{(\frac{-du}{dP})}{u} dP = \int dt$$

$$\frac{1}{a}\ln|P| - \frac{1}{a}\ln(a-bP) = t + C$$
$$\frac{1}{a}\ln|P| + (-\frac{1}{a}\ln|u|) = t + C$$
$$\frac{1}{a}\ln|P| - \frac{1}{a}\ln|a-bP| = t + C$$
$$\ln\frac{P}{a-bP} = at + aC$$
$$\frac{P}{a-bP} = Ce^{at}$$
$$P = Ce^{at}(a-bP)$$
$$P = aCe^{at} - bCe^{at}P$$
$$P + bCe^{at}P = aCe^{at}$$
$$P(1 + bCe^{at}) = aCe^{at}$$
$$P = \frac{aCe^{at}}{(1 + bCe^{at})}$$
$$P = \frac{aC}{bC + e^{-at}}$$

if $P(0) = P_0$, $P_0 \neq \frac{a}{b}$, $K = \frac{a}{b}$, $C = \frac{P_0}{a-bP}$ and $P_0 = C(a - bP)$, thus

$$P(t) = \frac{KP_0}{P_0 + (\frac{e^{-at}(a-bP_0)}{\frac{a}{K}})} \quad (4)$$

simplifying $\frac{e^{-at}(a-bP_0)}{\frac{a}{K}} = (K - P_0)e^{-at}$ and after substituting and simplifying the final $P(t)$ is as followed

$$P(t) = \frac{P_0 K}{P_0 + (K-P)e^{-at}} \quad (5)$$

***A.**Obtaining Equilibrium Points*

By setting the left-hand side of equation (1) to zero, the following system is in the form of steady state. Thus, it leads to the equilibrium points of the model. The trivial case is by substituting P = 0 and P = K. The equation (5) is then linearized in the neighborhood of its equilibrium point. Let assume $P(t) = P^* + \varepsilon(t)$, where $\varepsilon(t)$ is a small perturbation. Thus $\frac{d\varepsilon}{dt} = \frac{d}{dt}(P - P^*)$. By performing Taylor series expansion $\frac{dP}{dt} = f(P) = f(P^* + \varepsilon)$ the following power series is obtained

$$f(P^* + \varepsilon) = f(P^*) + \varepsilon\frac{df}{dt}|_{P=P^*} \quad (6)$$

since $f(P^*)$ is equal to zero and higher terms are assumed negligible, thus the approximate equation is $f(P^* + \varepsilon) = \varepsilon\frac{df}{dt}|_{P=P^*}$

$$\frac{df(P)}{dP} = aP(1-\frac{P}{K}) = aP - \frac{aP^2}{K} = a - \frac{2aP}{K} \quad (7)$$

substitute $P^* = 0$ and $P^* = K$ in $\frac{d\varepsilon}{dt}$ obtained

$$\frac{d\varepsilon}{dt} = \varepsilon(a - \frac{2aP}{K})|_{P=0} = a\varepsilon$$
$$\frac{d\varepsilon}{dt} = \varepsilon(a - \frac{2aP}{K})|_{P=K} = \varepsilon(a-2a) = -a\varepsilon.$$

According to Vikas and Shankar [7], if $f(P^*) > 0$, the equilibrium point P

1. is unstable if small perturbation $\varepsilon(t)$ grows exponentially
2. and stable if small perturbation $\varepsilon(t)$ decays exponentially.

Since a is a growth constant and $a > 0$, it indicates that the equilibrium point $P^* = 0$ is unstable as the perturbation $\varepsilon(t)$ grows exponentially while the equilibrium point $P^* = K$ is stable as the perturbation $\varepsilon(t)$ decays exponentially. This technique was applied to the palm oil optimal control model with the details shown by the following section.

II. STABILITY ANALYSIS OF PALM OIL MODEL

The dynamics between palm oil biomass and felling effect has a reciprocal relationship. Nasir *et al.*[9] has shown that if trees felled without control, less oil will be produced with less carbon to be absorbed. The stability of palm oil model with the presence of felling effect has been rarely discussed. Thus, following Vikas and Shankar [7] ideas and considering Gaoue *et al.* [8] and Chaudhary *et al.* [10] models. The optimal palm oil's control model with felling effect was examined for stability and analysed as follows.

A. *Model 1*

Model 1 has been previously developed, which considering biomass in representing the entire palm oil plantation. The underlying assumption of the model is that the biomass growth is influenced by logistic function. The logistic model was adjusted, which includes the felling activity that affects the production of fruit and carbon absorption. The model was created to help increasing the production of fresh fruit bunch. For the purpose of model analysis, the growth and felling rate are assumed to be constant. The model can be described as

$$\frac{dB}{dt} = aB(1-\frac{B}{K}) - cB \quad (8)$$

where B is defined as biomass plantation, a is denoted as growth rate and c represents as felling rate. The carrying capacity is denoted by $K = \frac{a}{b}$. Solving for $B(t) = bB[(K-\frac{c}{b}) - B$. The equilibrium points were obtained by finding all values of B that satisfy equation 8, $\frac{dB}{dt} = 0$

$$bB[(K-\frac{c}{b}) - B] = 0$$
$$bB[(K-\frac{c}{b}) - bB^2] = 0$$
$$bB(K-\frac{c}{b}) = bB^2$$
$$(K-\frac{c}{b}) = \frac{bB^2}{bB}$$
$$B = (K-\frac{c}{b})$$
$$B = \frac{a-c}{b}.$$

Thus, the two equilibrium points are $B = 0$ and $B = \frac{a-c}{b}$. To examine the stability of equilibrium point, consider a small perturbation around equilibrium point $B(t) = B^* + \varepsilon(t)$ where ε is a first order small quantity, such that

$$\frac{d\varepsilon}{dt} = \frac{d}{dt}(B - B^*)$$
$$\frac{dB}{dt} = f(B^* + \varepsilon). \quad (9)$$

Performing Taylor series expansion on equation (9)

$$f(B^*+\varepsilon) = f(B^*) + \varepsilon\frac{df}{dt}|_{B=B^*}$$
$$f(B^*+\varepsilon) = f(B^*) + \varepsilon\frac{df}{dt}|_{B=B^*} \quad (10)$$

since $f(B^*)$ is equal to zero and higher terms are assumed negligible, thus the approximate equation is

$$f(B^*+\varepsilon) = \varepsilon\frac{df}{dt}|_{B=B^*}$$

$$\frac{df(B)}{dB} = aB(1-\frac{B}{K}) - cB$$
$$\int\frac{df(B)}{dB} = \int[aB - \frac{aB^2}{K} - cB]dt \quad (11)$$
$$= a - \frac{2aB}{K} - c$$

substitute $B^* = 0$ and $B^* = \frac{a-c}{b}$ in $\frac{d\varepsilon}{dt}$ obtained $\varepsilon = \varepsilon_0 e^{(a-c)t}$ and $\varepsilon = \varepsilon_0 e^{(2c-a)t}$ respectively

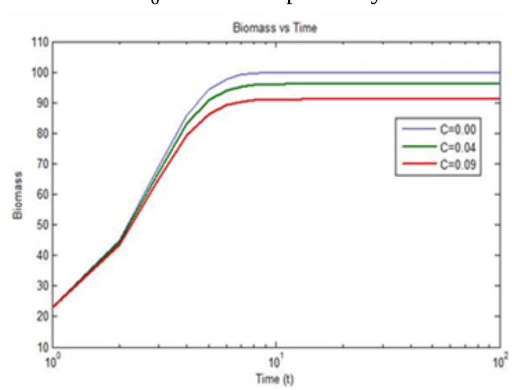

Figure 1: Graphs of biomass growth at different values of C

Plotting the biomass growth, $B(t) = \frac{B_0 K}{B_0 + (K - B_0)e^{-at}}$ as a function of time shows that B is approaching K. Since a is a growth constant and $a > 0$, it indicates that the equilibrium point $B^* = \frac{a-c}{b}$ is stable as the perturbation $\varepsilon(t)$ decays exponentially while the equilibrium point $B^* = 0$ is unstable as the perturbation $\varepsilon(t)$ grows exponentially . The result is illustrated in Figure 1 and 2 respectively.

B. *Model 2*

Model 2 is a continuous work of model 1, which is separating biomass into two parts, which are a young tree and mature tree. Similar to model 1, felling is the main contributor to increase the palm oil productivity. Besides felling rate, this model consider the important of re-planting new trees into the palm oil plantation.

This consideration is because the felling and planting give impact to the production of fresh fruit bunch and the absorption of carbon from the atmosphere. The model was seen able to increase the production of fresh fruit bunch and good in absorbing carbon from the atmosphere.

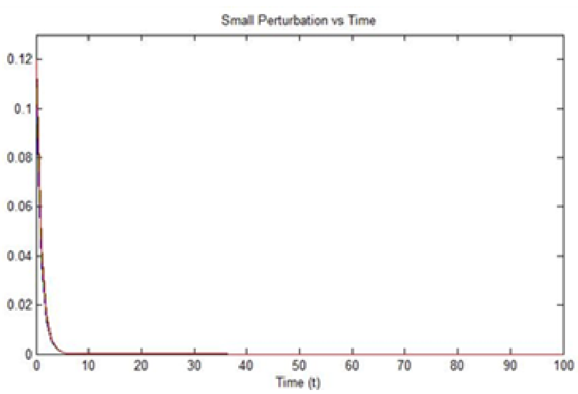

Figure 2: Graph of small perturbation of the equilibrium point

The analysis of stability was done to improve the performance of the model. The model is described as

$$\begin{aligned} \frac{dY}{dt} &= aY\left(1 - \frac{Y}{K}\right) - cY + nP \\ \frac{dP}{dt} &= cY - fP + mP \end{aligned} \qquad (12)$$

where Y is defined as young tree, P is mature tree, a represent growth rate, c represent transition rate from young to mature tree, n represent planting rate, f represent felling rate and m represent depletion rate. The carrying capacity is denotes by $K = \frac{a}{b}$. At the long-term scale, it is assume that the dynamics of P are at a quasi steady state in which $\frac{dP}{dt} = 0$, thus

$$\begin{aligned} cY - fP + mP &= 0 \\ cY &= fP - mP \\ P(f - m) &= cY \\ P &= \frac{cY}{f-m} \end{aligned} \qquad (13)$$

Simplifying equations (12) and (13) into the following single equation

$$\begin{aligned} \frac{dY}{dt} &= aY\left(1 - \frac{Y}{K}\right) - cY + n\left[\frac{cY}{f-m}\right] \\ &= aY\left(1 - \frac{Y}{K}\right) - cY + \frac{ncY}{f-m} \\ &= aY - \frac{aY^2}{K} - cY + \frac{ncY}{f-m} \end{aligned} \qquad (14)$$

the equilibrium points were obtained by finding all values of Y that satisfy equation 14 when $\frac{dY}{dt} = 0$

$$\begin{aligned} aY - \frac{aY^2}{K} - cY + \left(\frac{ncY}{f-m}\right) &= 0 \\ KbY - bY^2 &= cY - \left(\frac{ncY}{f-m}\right) \\ bY(K - bY) &= cY\left[1 - \left(\frac{n}{f-m}\right)\right] \\ &= \frac{cY}{bY}\left[1 - \left(\frac{n}{f-m}\right)\right] \\ &= \frac{c}{b}\left[1 - \left(\frac{n}{f-m}\right)\right] \\ bY &= K - \frac{c}{b}\left[1 - \left(\frac{n}{f-m}\right)\right] \\ Y &= \frac{K - \frac{c}{b}\left[1 - \left(\frac{n}{f-m}\right)\right]}{b} \\ Y &= \left[\frac{a}{b^2} - \frac{c}{b^2}\left[1 - \left(\frac{n}{f-m}\right)\right]\right] \end{aligned}$$

Thus, the two equilibrium points are $Y = 0$ and $Y = [\frac{a}{b^2} - \frac{c}{b^2}[1 - (\frac{n}{f-m})]]$. To examine the stability of equilibrium point, consider a small perturbation around equilibrium point $Y(t) = Y^* + \varepsilon(t)$ where ε is a first order small quantity, $\frac{d\varepsilon}{dt} = \frac{d}{dt}(Y - Y^*)$ such that $\frac{dY}{dt} = f(Y^* + \varepsilon)$. Performing Taylor series expansion on $\frac{dY}{dt}$ and assumed that the higher terms are negligible, thus the approximate equation is $f(Y^* + \varepsilon) = \varepsilon \frac{df}{dt}|_{Y=Y^*}$, substitute $Y^* = 0$ and $Y^* = [\frac{a}{b^2} - \frac{c}{b^2}[1 - (\frac{n}{f-m})]]$ into $\frac{d\varepsilon}{dt}$, obtained solution of $\varepsilon = \varepsilon_0 e^{(a-c)t}$ and $\varepsilon = \varepsilon_0 e^{(a - ([\frac{2a}{b} - \frac{2c}{b}[1 - (\frac{n}{f-m})]] - c)t}$ respectively.

Plotting $Y(t) = \frac{Y_0 Y^*}{Y_0 + (Y^* - Y_0)e^{-at}}$ as a function of time shows that Y is approaching K. Since a is a growth constant and $a > 0$, it indicates that the equilibrium point $Y^* = 0$ is unstable as the perturbation $\varepsilon(t)$ grows exponentially while the equilibrium point $Y^* = [\frac{2a}{b} - \frac{2c}{b}[1 - (\frac{n}{f-m})]]$ is stable as the perturbation $\varepsilon(t)$ decays exponentially. The result is illustrated in figure 3 and 4 respectively.

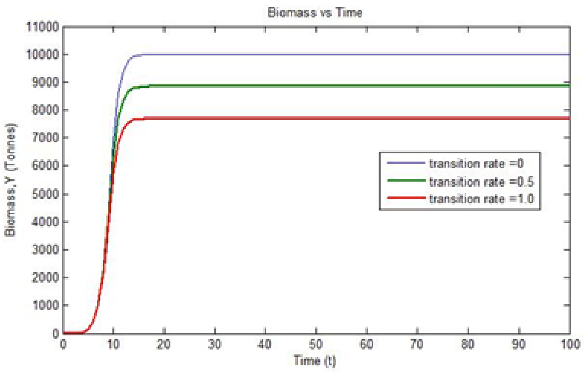

Figure 3: Graphs of biomass with different transition rate

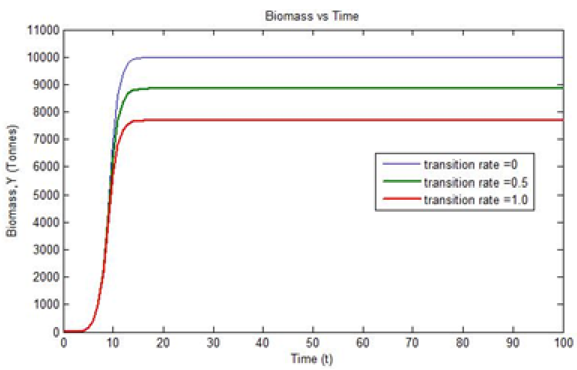

Figure 4: Graph of Small Perturbation (Y^*)

III. CONCLUSION

The analysis of system stability for palm oil plantation model has shown a positive result. Both models 1 and 2 have been analyzed using small perturbation in the neighborhood of equilibrium points. The equilibrium points of model 1 are $B^\wedge * = 0$ and $B^\wedge * = (a-c)/b$ while the equilibrium point of model 2 are $Y^\wedge * = 0$ and $Y^\wedge * = [2a/b - 2c/b[1-(n/(f-m))]]$ were identified. The criteria of the stability were established as the trajectories showed that small perturbation, $\varepsilon(t)$ for points $B^\wedge * = (a-c)/b$ and $Y^\wedge * = [2a/b - 2c/b[1-(n/(f-m))]]$ were decays exponentially as the time (t) is approaches infinity. The felling rate, plantation rate and the transition rate of young to mature tree did not affect the stability of the system.

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