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RELIABILITY MODELING AND OPTIMIZATION OF THE NUMBER OF HOT STANDBY UNITS IN A SYSTEM WORKING WITH TWO OPERATIVE UNITS

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Abstract: In the present paper, a reliability model is developed about the profit analysis and Optimization of number of hot standby units for a system working with two operative units. Hot standby works in a similar manner as operating unit means that when an operating unit fails, hot standby unit works with the same efficiency as the operating unit. The optimization of hot standby units is very important factor for any industry/unit/system for increasing the reliability redundancy and achieving the maximum profit. Thus, reliability models with no/one/two/three hot standby units in a system working with two operative units are developed. The cut-off points with regard to revenue, failure rate, etc. have been obtained to determine as to how many standby unit(s) should be there for the system. Comparative study has also been made to see which and when one of these models is better than the other as far as the profitability of the system is concerned. Semi-Markov processes and regenerative point technique have been used to obtain various performability measures.

Keywords: Two operative units, hot standby units, Regenerative point Technique, Profit analysis, Optimization

I. INTRODUCTION

Ranging from man to machine and in the present scenario also, technology has a great impact on every field of life. Due to increase in population and change in their tastes/ interests, demand of products is increasing continuously. To overcome this increasing demand, it is necessary to introduce the standby redundancy. Hot standby redundancy is that redundancy which is loaded with the same way as the operating unit when the operating unit fails. Many scholars have done a lot of work on hot standby units like Goel and Gupta (1983) discussed the analysis of a two-unit hot standby system with three modes. Christov and Stoytcheva (1999) dealt with the reliability and safety research of hot standby microcomputer signally systems. Rizwan et al. (2005) carried out the reliability analysis of a hot standby PLC system. Parashar and Taneja (2007) found the reliability and profit evaluation of a PLC hot standby system based on master-slave concept and two types of repair facilities. Rizwan et al. (2010) gave the reliability analysis of a hot standby industrial system. Kumar and Kumari (2017) carried out the comparative study of two-unit hot standby hardware software systems with impact of imperfect fault coverage. Manocha et al. (2017) discussed the stochastic and cost-benefit analysis of two-unit hot standby database system but the optimization of number of hot standby units for a system has not been taken into consideration by them. Batra and Taneja (2018) found a reliability model for the optimum number of hot standby units in a system working with one operative unit. However, there are many systems where systems comprising operative and hot standby system may require two operative units to meet out the demand. For such a system, working with two operative units, there is need to study as to how many hot standby units should be kept in order to get the optimum profit. To answer this question, we, in the present paper, develop four reliability models for a system having two operative units and:

i. No hot standby unit (Model 1)
ii. Two operative and one hot standby unit (Model 2)
iii. Two operative and two hot standby units (Model 3)
iv. Two operative and three hot standby units (Model 4).

The models are compared in order to optimize the number of hot standby units to be used. Analysis is done using semi-Markov processes and regenerative point technique.

II. Nomenclature

λ	Failure rate of operative unit
λ1	Failure rate of hot standby unit
g(t),G(t)	p.d.f. and c.d.f. of the repair time
Op	Operative unit
Hs	Standby unit
Fr	Failed unit under repair
Fwr	Failed unit is waiting for the repair
FR	Repair of the failed unit is continuing from previous state
C0	Revenue per unit up time
C1	Cost per unit up time for which the repairman is busy
C2	Cost per visit of the repairman
IC	Installation cost of an additional identical unit
Pi	Profit of model i; i=1,2,3,4
Φi(t)	C.d.f. of the first passage time from regenerative state i to a failed state
qij(t), Qij(t)	pdf, cdf of the first passage time from regenerative state Si to a regenerative state Sj
$AC_i^j(t)$	Probability that system working in full capacity at the instant t given that it entered Si at t=0 in case of model j; j=1, 2, 3, 4.
$B_i^j(t)$	Probability that the system is under repair at t given that the system entered Si at t=0 in case of j=1, 2, 3, 4.
$V_i^j(t)$	Expected number of visits in (0, t]; given that the system entered regenerative state Si at t=0 in case of j; j=1, 2, 3, 4.

III. Analysis of the Models

3.1 Model 1: System Comprising Two Operative Units and No Hot Standby Unit

In this model, we have considered a system wherein two units are operative and there is no hot standby unit. Possible transitions from one state to other are given as follows:

From	S_0	S_1	S_1	S_1	S_2
To	S_1	S_0	S_1	S_2	S_1
Via	...	...	S_2	...	...

where S_0 = (Op, Op), S_1= (Fr, Op), S_2 = (F_R, F_{wr}).
States S_0 and S_1 are regenerative states whereas S_2 is a non-regenerative state

3.1.1 Transition Probabilities and Mean Sojourn Times

The state transition probabilities $p_{ij} = \lim_{s\to 0} q_{ij}^*(s)$ can be obtained using the following: $q_{01}(t) = (2\lambda)e^{-(2\lambda)t}dt$,
$q_{10}(t) = e^{-\lambda t}g(t)dt$, $q_{12}(t) = \lambda e^{-\lambda t}\bar{G}(t)dt$,
$q_{11}^{(2)}(t) = (\lambda e^{-\lambda t}©1)g(t)dt$

Thus, we have

p_{01}=1, p_{10}=$g^*(\lambda)$, $p_{11}^{(2)}$=$g^*(0)$-$g^*(\lambda)$, p_{12}=$\lambda\bar{G}^*(\lambda)$

From these values, we have thefollowing relations

$p_{01} = 1$

$p_{10}+p_{12}=1$

$p_{10}+p_{11}^{(2)}=1$

Mean sojourn times (μ_i) i.e. the expected time of stay in regenerative state i are given as

$$\mu_0=\frac{1}{2\lambda},\ \ \mu_1=\frac{1-g^*(\lambda)}{\lambda}$$

Let $m_{ij}=\int_0^\infty tq_{ij}(t)dt= -q_{ij}{*}'(0)$,

i.e., $m_{01}=\mu_0$, $m_{10}+m_{12}=\mu_1$,

$$m_{10}+m_{11}^{(2)}=\int_0^\infty tg(t)dt = k_1(say)$$

3.1.2 Measures of System Effectiveness

3.1.2.1 Mean Time to System Failure (MTSF)

To determine the mean time to system failure (MTSF) of the system, we regard the failed state as absorbing state. Thus,

$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t)$

$\phi_1(t) = Q_{10}(t) \circledS \phi_0(t) + Q_{12}(t)$

Thus, $\mathrm{MTSF} = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N^1}{D^1}$, where

$N^1 = \mu_0 + \mu_1,\ D^1 = p_{12}$

3.1.2.2 *Availability*

The availability $AC_i(t)$ is seen to satisfy the following recursive relations:

$AC_0^1(t) = M_0^1(t) + q_{01}(t) © AC_1^1(t)$

$AC_1^1(t) = M_1^1(t) + q_{10}(t) © AC_0^1(t) + q_{11}^{(2)}(t) © AC_1^1(t)$

$M_0^1(t) = e^{-2\lambda t}$

$M_1^1(t) = e^{-\lambda t}\overline{G(t)}$

Taking Laplace Transforms and then solving the above equations for $AC_0^{1*}(s)$, the availability of the system, in steady state, is given by

$$AC_0^1 = \lim_{s\to 0} sAC_0^{1*}(s) = \frac{N_1^1}{D_1^1},$$

where

$N_1^1 = p_{10}\mu_0 + \mu_1,\ D_1^1 = p_{10}\mu_0 + k_1$

Proceeding in the similar manner as done in the case of obtaining expressions:

3.1.2.3 *Expected fraction of time during which the repairman is busy*

$$(B_0^1) = \lim_{s\to 0} sB_0^{1*}(s) = \lim_{s\to 0} \frac{sN_2^1(s)}{D_1^1(s)} = \frac{N_2^1}{D_1^1}$$

3.1.2.4 *Expected Number of Visits*

$$(V_0^1) = \lim_{s\to 0} sV_0^{1**}(s) = \lim_{s\to 0} \frac{sN_3^1(s)}{D_1^1(s)} = \frac{N_3^1}{D_1^1}$$

where

$N_2^1 = k_1$ $\qquad$ $N_3^1 = 1$

3.1.3 Profit Analysis

Profit equation in steady state is given by

Profit $(P_1) = C_0AC_0^1 - C_1B_0^1 - C_2V_0^1$

3.2 Model 2: System Comprising Two Operative Units and One Hot Standby Unit

In this model, system with two operative and one hot standby unit is considered. Possible transitions from one state to the other are shown as follows:

From	S_0	S_1	S_1	S_1	S_1	S_4	S_4
To	S_1	S_0	S_1	S_3	S_4	S_1	S_4
Via	...	...	S_2	S_2	S_2 and S_3	...	S_3

Where

S_0 = (Op, Hs, Hs), S_1= (Fr, Op, Hs), S_2 = (F_R, Fwr, Op), S_3 = (F_R, Fwr, Fwr), S_4= (Op, Fr, Fwr)

States S_0, S_1 and S_4 are regenerative states whereas S_2 and S_3 are non-regenerative states.

3.2.1 Transition Probabilities and Mean Sojourn Times

$q_{01}(t)=(2\lambda+\lambda_1)e^{-(2\lambda+\lambda_1)t}$, $q_{10}(t)=e^{-2\lambda t}g(t)$, $q_{11}^{(2)}(t)=(2\lambda e^{-2\lambda t} © e^{-\lambda t})g(t)$, $q_{13}^{(2)}(t)=(2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t})\overline{G}(t)$,

$q_{14}^{(2,3)}(t)=(2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t} © 1)g(t)$, $q_{41}(t)=e^{-\lambda t}g(t)dt$, $q_{44}^{(3)}(t)=(\lambda e^{-\lambda t} © 1)g(t)$

The transition probabilities $p_{ij} = \lim_{s\to 0} q_{ij}^*(s)$ for this model are obtained as

$p_{01} = 1, p_{10} = g^*(2\lambda),\ p_{11}^{(2)} = 2(g^*(\lambda) - g^*(2\lambda)),\ p_{13}^{(2)} = 2\lambda(\overline{G^*}(\lambda) - \overline{G^*}(2\lambda))$

$p_{14}^{(2,3)} = g^*(0) - 2g^*(\lambda) + g^*(2\lambda),\ p_{41} = g^*(\lambda),\ p_{44}^{(3)} = g^*(0) - g^*(\lambda)$

Thus, from these probabilities we conclude that

$p_{01}=1$

$p_{10}+p_{11}^{(2)}+p_{13}^{(2)}=1$

$p_{10}+p_{11}^{(2)}+p_{14}^{(2,3)}=1$

$p_{41}+p_{44}^{(3)}=1$

Mean Sojourn times (μ_i) for the model are:

$$\mu_0=\frac{1}{2\lambda+\lambda_1},\ \mu_1=\frac{1-g^*(2\lambda)}{2\lambda},\ \mu_4=\frac{1-g^*(\lambda)}{\lambda}$$

Here,

$m_{01}=\mu_0$

$m_{10}+m_{11}^{(2)}+m_{13}^{(2)}=\int_0^\infty t(e^{-2\lambda t}g(t)+2e^{-\lambda t}g(t)-2e^{-2\lambda t}g(t)+2\lambda e^{-\lambda t}\overline{G}(t)-2\lambda e^{-2\lambda t}\overline{G}(t))=k_2\,(\text{say})$

$m_{10}+m_{11}^{(2)}+m_{14}^{(2,3)}=\int_0^\infty tg(t)dt=k_1\,(\text{say}),\quad m_{41}+m_{44}^{(3)}=\int_0^\infty tg(t)dt=k_1\,(\text{say})$

3.2.2 Measures of System Effectiveness

3.2.2.1 Mean Time to System Failure (MTSF)

$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t)$

$\phi_1(t) = Q_{10}(t) \circledS \phi_0(t) + Q_{11}^{(2)}(t) \circledS \phi_1(t) + Q_{13}^{(2)}(t)$

Thus, $\mathrm{MTSF} = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} = \frac{N^2}{D^2}$, where

$N^2 = \mu_0\,(p_{10}+p_{13}^{(2)})+k_2$, $D^2 = p_{13}^{(2)}$

3.2.2.2 Availability at full Capacity

The availability $AC_i(t)$ is seen to satisfy the following recursive relations:

$$ACF_0^2(t) = M_0^2(t) + q_{01}(t)©ACF_1^2(t)$$

$$ACF_1^2(t) = M_{1f}^2(t) + q_{10}(t)©ACF_0^2(t) + q_{11}^{(2)}(t)©ACF_1^2(t) + q_{14}^{(2,3)}(t)©ACF_4^2(t)$$

$$ACF_2^2(t) = q_{41}(t)©ACF_1^2(t) + q_{44}^{(3)}(t)©ACF_4^2(t)$$

$$M_0^1(t) = e^{-(2\lambda+\lambda_1)t}$$

$$M_{1f}^1(t) = e^{-2\lambda t}\overline{G(t)}$$

Taking Laplace Transforms and then solving the above equations for $AC_0^{2*}(s)$, the availability of the system, in steady state, is given by

$$ACF_0^2 = \lim_{s\to 0} sACF_0^{2*}(s) = \frac{N_{1f}^2}{D_1^2},$$

Where,

$N_{1f}^2 = p_{41}p_{10}\mu_0 + p_{41}\mu_1$, $D_1^2 = p_{41}p_{10}\mu_0 + p_{14}^{(2,3)}\mu_3 + p_{41}k_2$

3.2.2.3 Availability at reduced Capacity

The availability $ACR_i(t)$ is seen to satisfy the following recursive relations:

$$ACR_0^2(t) = q_{01}(t)©ACR_1^2(t)$$

$$ACR_1^2(t) = M_{1r}^2(t) + q_{10}(t)©ACR_0^2(t) + q_{11}^{(2)}(t)©ACR_1^2(t) + q_{14}^{(2,3)}(t)©ACR_4^2(t)$$

$$ACR_2^2(t) = M_4^2(t) + q_{41}(t)©ACR_1^2(t) + q_{44}^{(3)}(t)©ACR_4^2(t)$$

$$M_{1r}^2(t) = (2\lambda e^{-2\lambda t}©e^{-\lambda t})\overline{G(t)} = k_3(\text{say})$$

$$M_4^2(t) = e^{-\lambda t}\overline{G(t)}$$

Taking Laplace Transforms and then solving the above equations for $ACR_0^{2*}(s)$, the availability of the system, in steady state, is given by

$$ACR_0^2 = \lim_{s\to 0} sACR_0^{2*}(s) = \frac{N_{1r}^2}{D_1^2}$$

, where

$N_{1r}^2 = p_{14}^{(2,3)}\mu_4 + p_{41}k_3$, $D_1^2 = p_{41}p_{10}\mu_0 + p_{14}^{(2,3)}\mu_3 + p_{41}k_2$

Proceeding in the similar manner as done in the case of obtaining expressions

3.2.2.4 Expected fraction of time during which the repairman is busy

$$(B_0^2) = \lim_{s\to 0} sB_0^{2*}(s) = \lim_{s\to 0}\frac{sN_2^2(s)}{D_1^2(s)} = \frac{N_2^2}{D_1^2}$$

3.2.2.5 Expected Number of Visits:

$$(V_0^2) = \lim_{s\to 0} sV_0^{2**}(s) = \lim_{s\to 0}\frac{sN_3^2(s)}{D_1^2(s)} = \frac{N_3^2}{D_1^2}$$

Where,

$N_2^2 = (p_{41} + p_{14}^{(2,3)})k_1$ and $N_3^2 = (1-p_{11}^{(2)})p_{41} + p_{14}^{(2,3)}(1+p_{41})$

3.2.3 Profit Analysis

Profit equation for standby unit in steady state is given by

Profit (P_2) = $C_0AC_0^2 - C_1B_0^2 - C_2V_0^2 - (IC_0)$

IC_0 is the installation cost of a hot standby unit per unit time.

3.3 Model 3: System Comprising Two Operative Units and Two Hot Standby Units

In this model, a system with two operative and two hot standby units have been considered. Possible state transitions are shown in the following table:

From	**S_0**	**S_1**	**S_1**	**S_1**	**S_1**	**S_1**
To	**S_1**	**S_0**	**S_1**	**S_4**	**S_5**	**S_6**
Via	**...**	**...**	**S_2**	**S_2 and S_3**	**S_2 and S_3**	**S_2, S_3 and S_5**
From	**S_4**	**S_4**	**S_4**	**S_4**	**S_6**	**S_6**
To	**S_1**	**S_4**	**S_5**	**S_6**	**S_4**	**S_6**
Via	**...**	**S_3**	**S_3**	**S_3 and S_5**		**S_5**

Where,

S_0 = (Op, Op, Hs, Hs), S_1= (Fr, Op, Op, Hs), S_2 = (F_R, Fwr, Op, Op), S_3 = (Op, F_R, Fwr, Fwr), S_4= (Op, Op, Fr, Fwr), S_5 = (F_R, Fwr, Fwr, Fwr), S_6 = (Op, F_r, Fwr, Fwr)

States S_0, S_1, S_4 and S_6 are regenerative states whereas S_2, S_3 and S_5 are non-regenerative states.

3.3.1 Transition Probabilities and Mean Sojourn Times

The transition probabilities are:

$q_{01}(1) = (2\lambda + 2\lambda_1)e^{-(2\lambda+2\lambda_1)t}dt$,

$q_{10}(t) = e^{-(2\lambda+\lambda_1)t}g(t)dt$,

$$q_{11}^{(2)}(t) = ((2\lambda+\lambda_1)©e^{-(2\lambda+\lambda_1)t}e^{-2\lambda t})g(t)dt$$
$$q_{14}^{(2,3)}(t) = ((2\lambda+\lambda_1)©e^{-(2\lambda+\lambda_1)t}\,2\lambda e^{-2\lambda t}©e^{-\lambda t})g(t)dt$$
$$q_{15}^{(2,3)}(t) = ((2\lambda+\lambda_1)e^{-(2\lambda+\lambda_1)t}©2\lambda e^{-2\lambda t}©e^{-\lambda t})\bar{G}(t)dt$$
$$q_{16}^{(2,3,5)}(t) = ((2\lambda+\lambda_1)e^{-(2\lambda+\lambda_1)t}©2\lambda e^{-2\lambda t}©\lambda e^{-\lambda t}©1)g(t)dt$$
$$q_{41}(t) = e^{-2\lambda t}g(t)dt\ ,\ q_{44}^{(3)}(t) = (2\lambda e^{-2\lambda t}©e^{-\lambda t})g(t)dt\ ,$$
$$q_{45}^{(3)}(t) = (2\lambda e^{-2\lambda t}©\lambda e^{-\lambda t})\bar{G}(t)dt$$
$$q_{46}^{(3,5)}(t) = (2\lambda e^{-2\lambda t}©\lambda e^{-\lambda t}©1)g(t)dt\ ,\ q_{64}(t) = e^{-\lambda t}g(t)dt$$
$$q_{66}^{(5)}(t) = (\lambda e^{-\lambda t}©1)g(t)dt$$

The transition probabilities are given as $p_{ij} = \lim_{s\to 0} q_{ij}^{*}(s)$

Here,

$$p_{01} = 1$$

$$p_{10} + p_{1}^{(2)} + p_{1}^{(2,3)} + p_{15}^{(2,3)} = 1$$

$$p_{10} + p_{11}^{(2)} + p_{14}^{(2,3)} + p_{16}^{(2,3,5)} = 1$$

$$p_{41} + p_{44}^{(3)} + p_{45}^{(3)} = 1$$

$$p_{41} + p_{44}^{(3)} + p_{46}^{(3,5)} = 1$$

$$p_{64} + p_{66}^{(5)} = 1$$

Mean Sojourn times (μ_i) for the model are:

$$\mu_0 = \frac{1}{2\lambda + 2\lambda_1},\ \mu_1 = \frac{1-g^*(2\lambda+\lambda_1)}{2\lambda+\lambda_1},\ \mu_4 = \frac{1-g^*(2\lambda)}{2\lambda},\ \mu_6 = \frac{1-g^*(\lambda)}{\lambda}$$

Thus,

$$m_{01} = \mu_0$$

$$m_{10} + m_{11}^{(2)} + m_{14}^{(2,3)} + m_{15}^{(2,3)} = \int_0^\infty t\left\{\frac{2\lambda^2(1-2\lambda.\lambda_1)}{\lambda_1(\lambda+\lambda_1)}e^{-(2\lambda+\lambda_1)}g(t) + \frac{(4\lambda^2+2\lambda\lambda_1-\lambda_1-2\lambda)}{\lambda_1}e^{-2\lambda t}g(t) + \frac{2(2\lambda+\lambda_1)(1-\lambda)}{(\lambda+\lambda_1)}e^{-\lambda t}g(t)\right\}$$

dt = K_5 (say)

$$m_{10} + m_{11}^{(2)} + m_{14}^{(2,3)} + m_{16}^{(2,3,5)} = \int_0^\infty tg(t)dt = k_1(\text{say})$$

$$m_{41} + m_{44}^{(3)} + m_{45}^{(3)} = \int_0^\infty t\{(2\lambda-1)e^{-2\lambda t} + 2(1-\lambda)e^{-\lambda t}\}g(t)dt = k_6(\text{say})$$

$$m_{41} + m_{44}^{(3)} + m_{46}^{(3,5)} = \int_0^\infty tg(t)dt = k_1(\text{say})$$

3.3.2 Measures of System Effectiveness

3.3.2.1 *Mean Time to System Failure (MTSF)*

$$Q_0(t) = Q_{01}(t)\circledS\ \phi_1(t)$$

$$Q_1(t) = Q_{10}(t)\circledS\ \phi_0(t)\ + Q_{11}^{(2)}(t)\circledS\ \phi_1(t)\ + Q_{14}^{(2,3)}(t)\circledS\ \phi_4(t)\ + Q_{15}^{(2,3)}(t)$$
$$Q_4(t) = Q_{41}(t)\circledS\ \phi_1(t)\ + Q_{44}^{(3)}(t)\circledS\ \phi_4(t)\ + Q_{45}^{(3)}(t)$$

MTSF when system starts from the state '0' is

$$MTSF = \lim_{s\to 0}\frac{1-\phi_0{**}(s)}{s}$$

$$= \lim_{s\to 0}\frac{D(s)-N(s)}{sD(s)} = \frac{'0'}{0}\text{form}$$

$$= \frac{D'(0)-N'(0)}{D(0)} = \frac{N^3}{D^3}$$

Where,

$$D^3 = p_{14}^{(2,3)}p_{45}^{(3)} + p_{15}^{(2,3)}(1-p_{44}^{(3)})$$

$$N^3 = p_{14}^{(2,3)}(m_{41}+m_{44}^{(3)}+m_{45}^{(3)}) + (p_{41}(p_{10}+p_{15}^{(2,3)}) + p_{45}^{(3)}(1-p_{11}^{(2)})\mu_0 + (p_{41}+p_{45}^{(3)})(m_{10}+m_{11}^{(2)}+m_{14}^{(2,3)}+m_{15}^{(2,3})$$
$$= p_{14}^{(2,3)}k_6 + (p_{41}(p_{10}+p_{15}^{(2,3)}) + p_{45}^{(3)}(1-p_{11}^{(2)}))\mu_0 + (p_{41}+p_{45}^{(3)})k_5$$

3.3.2.2 Availability at full Capacity

The availability $ACF_i(t)$ is seen to satisfy the following recursive relations:

$$ACF_0^3(t) = M_0^3(t) + q_{01}(t)©ACF_1^3(t)$$

$$ACF_1^3(t) = M_{1f}^3(t) + q_{10}(t)©ACF_0^3(t) + q_{11}^{(2)}(t)©ACF_1^3(t) + q_{14}^{(2,3)}(t)©ACF_4^3(t) + q_{16}^{(2,3,5)}(t)©ACF_6^3(t)$$
$$ACF_4^3(t) = M_{4f}^3(t) + q_{41}(t)©ACF_1^3(t) + q_{44}^{(3)}(t)©ACF_4^3(t) + q_{46}^{(3,5)}(t)©ACF_6^3(t)$$
$$ACF_6^3(t) = q_{64}(t)©ACF_4^3(t) + q_{66}^{(5)}©ACF_6^3(t)$$

Where,

$$M_0^3(t) = e^{-(2\lambda+2\lambda_1)t}$$
$$M_{1f}^3(t) = \frac{(2\lambda+\lambda_1)}{\lambda_1}e^{-2\lambda t}\bar{G}(t) - \frac{2\lambda}{\lambda_1}e^{-(2\lambda+\lambda_1)t}\bar{G}(t) = k_7(\text{say})$$
$$M_{4f}^3(t) = e^{-2\lambda t}\bar{G}(t)$$

Taking Laplace Transforms and then solving the above equations for $ACF_0^{3*}(s)$, the availability of the system, in steady state, is given by

$$ACF_0^3 = \lim_{s\to 0} sACF_0^3(t) = \lim_{s\to 0}\frac{sN_1(s)}{D_1(s)} = \frac{'0'}{0}\text{form}$$

$$= \lim_{s\to 0}\frac{sN_1'(s)+N_1(s)}{D_1'(s)} = \frac{N_1^3(0)}{D_1^3{}'(0)} = \frac{N_{1f}^3}{D_1^3}$$

Where,

$$N_{1f}^3 = p_{64}p_{41}p_{10}\mu_0 + p_{64}(1-p_{10}-p_{11}^{(2)})\mu_4 + p_{64}p_{41}k_7$$

$$D_1^3 = p_{64}p_{41}(m_{10} + m_{11}^{(2)} + m_{14}^{(2,3)} + m_{16}^{(2,3,5)}) + p_{64}(p_{14}^{(2,3)} + p_{16}^{(2,3,5)})(m_{41} + m_{44}^{(3)} + m_{46}^{(3,5)})$$
$$+ \; p_{64}p_{41}p_{10}\mu_0 + \mu_5(p_{41}p_{16}^{(2,3,5)} + p_{46}^{(3,5)}(p_{14}^{(2,3)} + p_{16}^{(2,3,5)})$$

$$\Rightarrow \; p_{64}p_{41}p_{10}\mu_0 + \{p_{41}(p_{16}^{(2,3,5)} + p_{64}) + (p_{46}^{(3,5)} + p_{64})(p_{14}^{(2,3)} + p_{16}^{(2,3,5)})\}k_1$$

3.3.2.2 *Availability at reduced Capacity*

The availability $ACR_i(t)$ is seen to satisfy the following recursive relations:

$$ACR_0^3(t) = q_{01}(t)©ACR_1^3(t)$$

$$ACR_1^3(t) = M_{1r}^3(t) + q_{10}(t)©ACR_0^3(t) + q_{11}^{(2)}(t)©ACR_1^3(t) + q_{14}^{(2,3)}(t)©ACR_4^3(t)$$
$$+q_{16}^{(2,3,5)}©ACR_6^3(t)$$

$$ACR_4^3(t) = M_{4r}^3(t) + q_{41}(t)©ACR_1^3(t) + q_{44}^{(3)}(t)©ACR_4^3(t) + q_{46}^{(3,5)}(t)©ACR_6^3(t)$$
$$ACR_6^3(t) = M_6^3(t) + q_{64}(t)©ACR_4^3(t) + q_{66}^{(5)}(t)©ACR_6^3(t)$$

Where,

$$M_{1r}^3(t) = \frac{2(2\lambda+\lambda_1)}{\lambda+\lambda_1}e^{-\lambda t}\bar{G}(t) + \frac{2(2\lambda+\lambda_1)}{\lambda_1}e^{-2\lambda t}\bar{G}(t) - \frac{2(2\lambda+\lambda_1)(\lambda+2\lambda_1)}{\lambda_1(\lambda+\lambda_1)}e^{-(2\lambda+\lambda_1)t}\bar{G}(t) = k_8(\text{say})$$
$$M_{4r}^3(t) = 2e^{-\lambda t}\bar{G}(t) - 2e^{-2\lambda t}\bar{G}(t) = k_9(\text{say})$$
$$M_6^3(t) = e^{-\lambda t}\bar{G}(t)$$

Taking Laplace Transforms and then solving the above equations for $ACR_0^{3*}(s)$, the availability of the system, in steady state, is given by:

$$ACR_0^3 = \lim_{s\to 0} sACR_0^3(t) = \lim_{s\to 0}\frac{sN_1(s)}{D_1(s)} = \frac{'0'}{0}\text{form}$$

$$= \lim_{s\to 0}\frac{sN_1'(s) + N_1(s)}{D_1'(s)} = \frac{N_1^3(0)}{D_1^3{}'(0)} = \frac{N_{1r}^3}{D_1^3}$$

Where,

$$N_{1r}^3 = p_{64}p_{41}k_8 + (p_{41}p_{16}^{(2,3,5)} + p_{46}^{(3,5)}(p_{14}^{(2,3)} + p_{16}^{(2,3,5)}))\mu_6$$
$$+p_{64}(p_{14}^{(2,3)} + p_{16}^{(2,3,5)})k_9$$ and D_1^3 is already defined.

Proceeding in the similar manner as done in the case of obtaining expressions

3.3.2.4 Expected fraction of time during which the repairman is busy

$$(B_0^3) = \lim_{s\to 0} sB_0^{3*}(s) = \lim_{s\to 0}\frac{sN_2^3(s)}{D_1^3(s)} = \frac{N_2^3}{D_1^3}$$

3.3.2.5 Expected Number of Visits

$$(V_0^3) = \lim_{s\to 0} sV_0^{3**}(s) = \lim_{s\to 0}\frac{sN_3^3(s)}{D_1^3(s)} = \frac{N_3^3}{D_1^3}$$

Where,

$$N_2^3 = (p_{64}(p_{41} + p_{16}^{(2,3,5)}) + (p_{46}^{(3,5)} + p_{64})(p_{14}^{(2,3)} + p_{16}^{(2,3,5)}))k_1$$ and
$$N_3^3 = p_{64}p_{41}p_{10}$$

3.3.4 Profit Analysis

Profit equation for two standby units in steady state is given by:

Profit $(P_3) = C_0AC_0^3 - C_1B_0^3 - C_2V_0^3 - 2(IC_0)$

3.4 Model 4: System having Two Operative Units and Three Hot Standby Units:

In this model, we have considered a system wherein two units are operative and three hot standby units which take place of the operative unit if the latter gets failed. Possible transitions from one state to other one given as follows:

From	S_0	S_1	S_1	S_1	S_1	S_1	S_1	S_6	S_6	S_7
To	S_1	S_0	S_1	S_5	S_7	S_8	S_6	S_7	S_6	S_7
Via	...	...	S_2	S_2, S_3 and S_4	S_2, S_3 and S_4	S_2 and S_3	S_2, S_3, $S_4$and S_5	...	S_5	S_4
From	S_7	S_7	S_7	S_8	S_8	S_8	S_7	S_8	S_8	
To	S_8	S_6	S_5	S_1	S_8	S_7	S_5	S_5	S_6	
Via		S_4 and S_5	S_4		S_3	S_3 and S_4	S_4	S_3 and S_4	S_3, S_4 and S_5	

Where,

S_0 = (Op, Op, Hs, Hs, Hs), S_1= (Fr, Op, Op, Hs, Hs), S_2 = (F_R, Fwr, Hs, Op, Op), S_3 = (Op, Op, F_R, Fwr, Fwr), S_4= (Op, F_R , Fwr, Fwr, Fwr), S_5 = (F_R, Fwr, Fwr, Fwr, Fwr), S_6 = (Op, F_r, Fwr, Fwr, Fwr), S_7 = (Op, O_p, Fwr, Fwr, Fr), S_6 = (Op, O_p, Hs, Fwr, Fr)

States S_0, S_1, S_6, S_7 andS_8 are regenerative states whereas S_2, S_3, S_4 and S_5 are non-regenerative states.

3.4.1 Transition Probabilities and Mean Sojourn Times

The transition probabilities are:

$$q_{01}(1) = (2\lambda + 3\lambda_1)e^{-(2\lambda+3\lambda_1)t}dt \;,\; q_{10}(t) = e^{-(2\lambda+2\lambda_1)t}g(t)dt$$

$$q_{11}^{(2)}(t) = ((2\lambda + 2\lambda_1)e^{-(2\lambda+2\lambda_1)t}©e^{-(2\lambda+\lambda_1)t})g(t)dt$$

$$q_{15}^{(2,3,4)}(t) = ((2\lambda + 2\lambda_1)e^{-(2\lambda+2\lambda_1)t}©(2\lambda + \lambda_1)e^{-(2\lambda+\lambda_1)t}©2\lambda e^{-2\lambda t}©\lambda e^{-\lambda t})\bar{G}(t)dt$$
$$q_{18}^{(2,3)}(t) = ((2\lambda + 2\lambda_1)e^{-(2\lambda+2\lambda_1)t}©(2\lambda + \lambda_1)e^{-(2\lambda+\lambda_1)t}©e^{-2\lambda t})g(t)dt$$

$$q_{17}^{(2,3,4)}(t) = ((2\lambda + 2\lambda_1)e^{-(2\lambda+2\lambda_1)t}©(2\lambda + \lambda_1)e^{-(2\lambda+\lambda_1)t}©2\lambda e^{-2\lambda t}©e^{-\lambda t})g(t)dt$$
$$q_{16}^{(2,3,4,5)}(t) = ((2\lambda + 2\lambda_1)e^{-(2\lambda+2\lambda_1)t}©(2\lambda + \lambda_1)e^{-(2\lambda+\lambda_1)t}©2\lambda e^{-2\lambda t}©e^{-\lambda t}©1)g(t)dt$$

$q_{67}(t) = e^{-\lambda t} g(t)dt$, $q_{66}^{(5)}(t) = (2\lambda e^{-\lambda t} © 1) g(t)dt$,

$q_{78}(t) = e^{-2\lambda t} g(t)dt$

$q_{77}^{(4)}(t) = (2\lambda e^{-2\lambda t} © e^{-\lambda t}) g(t)dt$,

$q_{76}^{(4,5)}(t) = (2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t} © 1) g(t)dt$

$q_{81}(t) = e^{-2\lambda t} g(t)dt$, $q_{88}^{(3)}(t) = (2\lambda e^{-2\lambda t} © e^{-2\lambda t}) g(t)dt$

$q_{87}^{(3,4)}(t) = (2\lambda e^{-2\lambda t} © 2\lambda e^{-2\lambda t} © e^{-\lambda t}) g(t)dt$

$q_{86}^{(3,4,5)}(t) = (2\lambda e^{-2\lambda t} © 2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t} © 1) g(t)dt$

$q_{75}^{(4)}(t) = (2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t}) \bar{G}(t)dt$

$q_{85}^{(3,4)}(t) = (2\lambda e^{-2\lambda t} © 2\lambda e^{-2\lambda t} © \lambda e^{-\lambda t}) \bar{G}(t)dt$

It can be checked that

$p_{01} = 1$

$p_{10} + p_{11}^{(2)} + p_{15}^{(2,3,4)} + p_{18}^{(2,3)} + p_{17}^{(2,3,4)} = 1$

$p_{10} + p_{11}^{(2)} + p_{18}^{(2,3)} + p_{17}^{(2,3,4)} + p_{16}^{(2,3,4,5)} = 1$

$p_{64} + p_{66}^{(5)} = 1$

$p_{78} + p_{77}^{(4)} + p_{76}^{(4,5)} = 1$

$p_{78} + p_{77}^{(4)} + p_{75}^{(4)} = 1$

$p_{81} + p_{88}^{(3)} + p_{87}^{(3,4)} + p_{85}^{(3,4)} = 1$

$p_{81} + p_{88}^{(3)} + p_{87}^{(3,4)} + p_{86}^{(3,4,5)} = 1$

Mean Sojourn times (μ_i) for the model are:

$$\mu_0 = \frac{1}{2\lambda + 3\lambda_1},\ \mu_1 = \frac{1 - g^*(2\lambda + 2\lambda_1)}{2\lambda + 2\lambda_1},\ \mu_6 = \frac{1 - g^*(\lambda)}{\lambda},\ \mu_7 = \frac{1 - g^*(2\lambda)}{2\lambda} = \mu_8$$

Here,

$m_{01} = \mu_0$

$$\begin{aligned} m_{10} + m_{11}^{(2)} + m_{18}^{(2,3)} + m_{17}^{(2,3,4)} + m_{15}^{(2,3,4)} = \int_0^\infty t\{ & e^{-(2\lambda+2\lambda_1)t} g_1(t) + \frac{2(\lambda+\lambda_1)}{\lambda_1}(e^{-(2\lambda+\lambda_1)t} - e^{-(2\lambda+2\lambda_1)t}) g(t) \\ & + \frac{(\lambda+\lambda_1)(2\lambda+\lambda_1)}{\lambda_1^2}(e^{-(2\lambda)t} - 2e^{-(2\lambda+\lambda_1)t} + e^{-(2\lambda+2\lambda_1)t}) g(t) \\ & + 4\lambda(\lambda+\lambda_1)(2\lambda+\lambda_1)(\frac{e^{-\lambda t}}{\lambda(\lambda+\lambda_1)(2\lambda+\lambda_1)} - \frac{e^{-2\lambda t}}{2\lambda\lambda_1^2} \\ & + \frac{e^{-(2\lambda+\lambda_1)t}}{\lambda_1^2(\lambda+\lambda_1)} + \frac{e^{-(2\lambda+2\lambda_1)t}}{2\lambda_1^2(\lambda+2\lambda_1)}) g(t) \\ & + 4\lambda^2(\lambda+\lambda_1)(2\lambda+\lambda_1)(\frac{e^{-\lambda t}}{\lambda(\lambda+\lambda_1)(2\lambda+\lambda_1)} - \frac{e^{-2\lambda t}}{2\lambda\lambda_1^2} \\ & + \frac{e^{-(2\lambda+\lambda_1)t}}{\lambda_1^2(\lambda+\lambda_1)} + \frac{e^{-(2\lambda+2\lambda_1)t}}{2\lambda_1^2(\lambda+2\lambda_1)}) \bar{G}(t)\} dt = k_{10}(say) \end{aligned}$$

$$m_{10} + m_{11}^{(2)} + m_{18}^{(2,3)} + m_{16}^{(2,3,4,5)} + m_{17}^{(2,3,4)} = \int_0^\infty t g(t)dt = k_1(say)$$

$$m_{67} + m_{66}^{(5)} = \int_0^\infty t g(t)dt = k_1$$

$$m_{78} + m_{77}^{(4)} + m_{75}^{(4)} = \int_0^\infty t\{e^{-2\lambda t} + 2(e^{-\lambda t} - e^{-2\lambda t}) + 2\lambda(e^{-\lambda t} - e^{-2\lambda t})\} g(t)dt = k_{11}(say)$$

$$m_{78} + m_{77}^{(4)} + m_{76}^{(4,5)} = \int_0^\infty t g(t)dt = k_1(say)$$

$$m_{81} + m_{88}^{(3)} + m_{87}^{(3,4)} + m_{85}^{(3,4)} = \int_0^\infty \{e^{-2\lambda t} + 2\lambda t e^{-2\lambda t} + 4(e^{-\lambda t} - e^{-2\lambda t} - \lambda t e^{-2\lambda t})\} g(t)dt + 4\lambda(e^{-\lambda t} - e^{-2\lambda t} - \lambda t e^{-2\lambda t}) \bar{G}(t)dt = k_{12}(say)$$

$$m_{81} + m_{88}^{(3)} + m_{87}^{(3,4)} + m_{86}^{(3,4,5)} = \int_0^\infty t g(t)dt = k_1$$

4.4.2 Measures of System Effectiveness

3.4.2.1 Mean Time to System Failure (MTSF)

$\phi_0(t) - Q_{01}(t) \circledS \phi_1(t) = 0$

$Q_{10}^{(2)}\phi_0(t) - \phi_1(t)(1 - q_{11}^{(2)}) - q_{17}^{(2,3,4)}(t)\phi_7 - q_{18}^{(2,3)}\phi_8 = q_{15}^{(2,3,4)}$

$(1 - q_{77}^{(4)})\phi_7 - q_{78}\phi_8 = q_{75}^{(4)}$

$-q_{81}\phi_1 - q_{87}^{(3,4)}\phi_7 - q_{88}^{(3)}\phi_8 = q_{85}^{(3,4)}$

$$\text{MTSF} = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s}$$

$$= \lim_{s \to 0} \frac{1 - \frac{N(s)}{D(s)}}{s} = \frac{D'(0) - N'(0)}{D(0)} = \frac{N^4}{D^4}$$

Where,

$$\begin{aligned} N^4 = & \{p_{78}p_{17}^{(2,3,4)} + p_{18}^{(2,3)}(p_{78} + p_{75}^{(4)}\}\mu_5 + \{p_{17}^{(2,3,4)}(p_{81} + p_{85}^{(3,4)} + p_{87}^{(3,4)}) + p_{18}^{(2,3)}p_{87}^{(3,4)}\}k_1 \\ & + \{p_{78}p_{81} + p_{75}^{(4)}\{p_{81} + p_{87}^{(3,4)} + p_{85}^{(3,4)}\}k_3 \\ & + [\{p_{78}(p_{81}p_{85}^{(3,4)}) + p_{75}^{(4)}(p_{81} + p_{87}^{(3,4)})p_{85}^{(3,4)})\}(p_{01} + p_{15}^{(2,3,4)})]\mu_0 \end{aligned}$$

$$\begin{aligned} D^4 = & (1 - p_{11}^{(2)} - p_{10})[(1 - p_{77}^{(4)})(1 - p_{88}^{(3)}) - p_{78}p_{87}^{(3,4)}] \\ & - p_{81}[p_{78}p_{17}^{(2,3,4)} + p_{18}^{(2,3)}(1 - p_{77}^{(4)})] \end{aligned}$$

3.4.2.2 Availability at full Capacity

The availability $ACF_i(t)$ is seen to satisfy the following recursive relations:

$$ACF_0^4(t) = M_0^4(t) + q_{01}(t) © ACF_1^4(t)$$

$$\begin{aligned} ACF_1^4(t) = & M_{1f}^4(t) + q_{10}(t) © ACF_0^4(t) + q_{11}^{(2)}(t) © ACF_1^4(t) + q_{16}^{(2,3,4,5)}(t) © ACF_6^4(t) \\ & + q_{17}^{(2,3,4)}(t) © ACF_7^4(t) + q_{18}^{(2,3)}(t) © ACF_8^4(t) \end{aligned}$$

$$ACF_6^4(t) = q_{67}(t) © ACF_7^4(t) + q_{66}^{(5)}(t) © ACF_6^4(t)$$

$$ACF_8^4(t) = M_{8f}^4(t) + q_{81}(t) © ACF_1^4(t) + q_{88}^{(3)}(t) © ACF_8^4(t) + q_{87}^{(3,4)}(t) © ACF_7^4(t) + q_{86}^{(3,4,5)}(t) © ACF_6^4(t)$$

Thus,

$$ACF_0^4 = \lim_{s \to 0} s ACF_0^4 * (s) = \frac{N_{1f}^4}{D_1^4}$$

$$\begin{aligned} N_{1f}^4 = & p_{67}p_{78}p_{81}(1 - p_{11}^{(2)} + p_{10}k_{12})\mu_0 - (p_{81}\mu_0 - p_{01}k_{13})(p_{18}^{(2,3)} + p_{17}^{(2,3,4)} + p_{16}^{(2,3,4,5)}) \\ & + p_{01}\mu_7\{(p_{81}p_{67}(p_{17}^{(2,3,4)} + p_{16}^{(2,3,4,5)}) + p_{67}(p_{16}^{(2,3,4,5)} + p_{17}^{(2,3,4)} + p_{18}^{(2,3)})(p_{87}^{(3,4)} + p_{86}^{(3,4,5)})\} \end{aligned}$$

$$D_1^4 = \{(p_{16}^{(2,3,4,5)} + p_{17}^{(2,3,4)} + p_{18}^{(2,3)})(p_{67}p_{78} + p_{67}p_{87}^{(3,4)} + p_{67}p_{86}^{(3,4,5)}p_{76}^{(4,5)}(1-p_{88}^{(3)}) + p_{78}p_{86}^{(3,4,5)})$$
$$+p_{67}p_{78}p_{81} + p_{67}p_{81}(p_{17}^{(2,3,4)} + p_{16}^{(2,3,4,5)}) +$$
$$p_{81}(p_{16}^{(2,3,4,5)}p_{78} - p_{18}^{(2,3)}p_{76}^{(4,5)})\}k_4 + p_{67}p_{78}p_{81}p_{10}\mu_0$$

3.4.2.3 Availability at reduced Capacity

The availability $ACR_i(t)$ is seen to satisfy the following recursive relations:

$$ACR_0^4(t) = M_0^4(t) + q_{01}(t)©ACR_1^4(t)$$

$$ACR_1^4(t) = M_{1f}^4(t) + q_{10}(t)©ACR_0^4(t) + q_{11}^{(2)}(t)©ACR_1^4(t) + q_{16}^{(2,3,4,5)}(t)©ACR_6^4(t)$$
$$+q_{17}^{(2,3,4)}(t)©ACR_7^4(t) + q_{18}^{(2,3)}(t)©ACR_8^4(t)$$

$$ACR_6^4(t) = q_{67}(t)©ACR_7^4(t) + q_{66}^{(5)}(t)©ACR_6^4(t)$$

$$ACR_8^4(t) = M_{8f}^4(t) + q_{81}(t)©ACR_1^4(t) + q_{88}^{(3)}(t)©ACR_8^4(t) + q_{87}^{(3,4)}(t)©ACR_7^4(t) + q_{86}^{(3,4,5)}(t)©ACR_6^4(t)$$

Thus,

$$ACR_0^4 = \lim_{s\to 0} sACR_0^4{*}(s) = \frac{N_{1f}^4}{D_1^4}$$

$$N_{1r}^4 = p_{67}p_{78}p_{81}k_{14} + \mu_6\{p_{78}p_{81}p_{16}^{(2,3,4,5)} + p_{78}p_{86}^{(3,4,5)}(p_{16}^{(2,3,4,5)} + p_{10} + p_{18}^{(2,3)})$$
$$+p_{76}^{(4,5)}(p_{81} + p_{87}^{(3,4)} + p_{86}^{(3,4,5)})(p_{16}^{(2,3,4,5)} + p_{17}^{(2,3,4)}) + p_{76}^{(4,5)}p_{18}^{(2,3)}p_{86}^{(3,4,5)} + p_{18}^{(2,3)}p_{87}^{(3,4)}p_{76}^{(3,4)}\}$$
$$+k_{15}\{p_{67}(1-p_{88}^{(3)})(p_1^{(2,3,4,5)} + p_{17}^{(2,3,4)}) + p_{67}p_{18}^{(2,3)}(p_{87}^{(3,4)} + p_{86}^{(3,4,5)})\} + k_{16}p_{67}p_{78}(p_{16}^{(2,3,4,5)} + p_{17}^{(2,3,4)} + p_{18}^{(2,3)})$$

Proceeding in the similar manner as done in the case of obtaining expressions:

3.4.2.4 Expected fraction of time during which the repairman is busy:

$$(B_0^4) = \lim_{s\to 0} sB_0^{4*}(s) = \lim_{s\to 0}\frac{sN_2^4(s)}{D_1^4(s)} = \frac{N_2^4}{D_1^4}$$

3.4.2.5 Expected Number of Visits

$$(V_0^4) = \lim_{s\to 0} sV_0^{4**}(s) = \lim_{s\to 0}\frac{sN_3^4(s)}{D_1^4(s)} = \frac{N_3^4}{D_1^4}$$

Where,

$$N_2^4 = [p_{67}p_{78}p_{81} + p_{16}^{(2,3,4,5)}((p_{81} + p_{86}^{(3,4,5)})(p_{78} + p_{76}^{(4,5)}) + p_{76}^{(4,5)}p_{87}^{(3,4)}) + p_{17}^{(2,3,4)}(-p_{86}^{(3,4,5)}(p_{78} + p_{76}^{(4,5)})$$
$$-p_{76}^{(4,5)}(p_{81} + p_{87}^{(3,4)})) - p_{18}^{(2,3)}(p_{76}^{(4,5)}p_{87}^{(3,4)} + p_{86}^{(3,4,5)}(p_{78} + p_{76}^{(4,5)}))$$
$$+p_{67}(p_{87}^{(3,4)}\{p_{67}p_{78}(p_{16}^{(2,3,4,5}) + p_{17}^{(2,3,4)} + p_{18}^{(2,3)}\}$$
$$+(p_{86}^{(3,4,5)})(p_{16}^{(2,3,4,5)} + p_{17}^{(2,3,4)} + p_{18}^{(2,3)}) + p_{67}p_{81}(p_{16}^{(2,3,4,5)} + p_{17}^{(2,3,4)})]k_1$$

And $N_3^4 = p_{67}p_{78}p_{81}p_{10}$

3.4.3 Profit Analysis

Profit equation for three standby in steady state is given by
Profit $(P_4) = C_0AC_0^4 - C_1B_0^4 - C_2V_0^4 - 3(IC_0)$

IV. Comparative Study among the Models

4.1 Optimization of Number of Hot Standby Units with regard to Revenue per Unit up Time:

a) On comparing the profits of Models 1 and 2, we conclude that Model 1 is better or worse than Model 2

if P_1-P_2 >0 or < 0

i.e. if $(C_0AC_0^1 - C_1B_0^1 - C_2V_0^1)$-$(C_0AC_0^2 - C_1B_0^2 - C_2V_0^2$-$(IC_0))$ >0 or <0 i.e. if $\begin{Bmatrix} C_0 > \text{or} < C_{01}^* \text{for} AC_0^1 > AC_0^2 \\ C_0 < \text{or} > C_{01}^* \text{for} AC_0^1 < AC_0^2 \end{Bmatrix}$

Both the models are equally good if $C_0 = C_{01}^*$.

$$\text{where } C_{01}^* = \frac{(C_1(B_0^1 - B_0^2) + C_2(V_0^1 - V_0^2) - IC_0)}{(AC_0^1 - AC_0^2)}$$

b) Comparison between Models 2 and 3 reveals that Model 2 is better or worse than Model 3

if P_2-P_3 >0 or < 0

i.e. if $(C_0AC_0^2 - C_1B_0^2 - C_2V_0^2$-$(IC_0))$-$(C_0AC_0^3 - C_1B_0^3 - C_2V_0^3$-$2*(IC_0))$ >0 or <0

i.e. if $\begin{Bmatrix} C_0 > \text{or} < C_{02}^* \text{ for } AC_0^2 > AC_0^3 \\ C_0 < \text{or} > C_{02}^* \text{ for } AC_0^2 < AC_0^3 \end{Bmatrix}$

Both are equally good if $C_0 = C_{02}^*$.

$$\text{where } C_{02}^* = \frac{C_1(B_0^2 - B_0^3) + C_2(V_0^2 - V_0^3) - IC_0}{(AC_0^2 - AC_0^3)}$$

c) As far as the selection between Model 3 and 1 is concerned, one should adopt Model 3 in preference to Model 1:

if P_3-P_1 >0 or < 0

i.e. if $(C_0AC_0^3 - C_1B_0^3 - C_2V_0^3$-$2*(IC_0))$-$(C_0AC_0^1 - C_1B_0^1 - C_2V_0^1)$ > 0 or < 0

i.e. if $\begin{Bmatrix} C_0 > \text{or} < C_{03}^*, \text{for } AC_0^3 > AC_0^1 \\ C_0 < \text{or} > C_{03}^*, \text{for } AC_0^3 < AC_0^1 \end{Bmatrix}$

Both are equally good if $C_0 = C_{03}^*$.

$$\text{where } C_{03}^* = \frac{C_1(B_0^3 - B_0^1) + C_2(V_0^3 - V_0^1) + 2*(IC_0)}{(AC_0^3 - AC_0^1)}$$

4.2 Optimization of Number of Hot Standby Units with regard to Cost of Installing a Hot Standby Unit:

a) On comparing thr profits of Model 1 and 2, we conclude that Model 1 is better or worse than Model 2

if P_1-P_2 >0 or <0

i.e. if $(C_0AC_0^1 - C_1B_0^1 - C_2V_0^1$-$IC_0)$-$(C_0AC_0^2 - C_1B_0^2 - C_2V_0^2$-$2*(IC_0))$ >0 or <0

i.e. if IC_0 > or < IC_{01}^*

Both are equally good if $IC_0 = IC_{01}^*$

Where, IC_{01}^{*} = $-C_0(AC_0^1 - AC_0^2) + C_1(B_0^1 - B_0^2) + C_2(V_0^1 - V_0^2)$ **c)**

b) Comparing between Model 2 and 3 reveals that Model 2 is better or worse than Model 3

if $P_2 - P_3 > 0$ or < 0

i.e. if $(C_0AC_0^2 - C_1B_0^2 - C_2V_0^2 - (IC_0)) - (C_0AC_0^3 - C_1B_0^3 - C_2V_0^3 - 2*(IC_0)) > 0$ or < 0

i.e. if $IC_0 >$ or $< IC_{02}^{*}$

Both are equally good if $IC_0 = IC_{02}^{*}$

Where, $IC_{02}^{*} = -C_0(AC_0^2 - AC_0^3) + C_1(B_0^2 - B_0^3) + C_2(V_0^2 - V_0^3)$

c) As far as the selection between **Model 3 and 1** is concerned, one should adopt **Model 3** in preference to **Model 1**

if $P_3 - P_1 > 0$ or < 0

i.e. if $(C_0AC_0^3 - C_1B_0^3 - C_2V_0^3 - 2*(IC_0)) - (C_0AC_0^1 - C_1B_0^1 - C_2V_0^1) > 0$ or < 0

i.e. if $IC_0 >$ or $< IC_{03}^{*}$

Both are equally good if $IC_0 = IC_{03}^{*}$

Where, $IC_{03}^{*} = \frac{C_0(AC_0^3 - AC_0^1) - C_1(B_0^3 - B_0^1) - C_2(V_0^3 - V_0^1)}{2}$

4.3 Optimization of Number of Hot Standby Units with regard to Cost per visit of the repairman:

a) On comparing thr profits of **Model 1 and 2**, we conclude that **Model 1** is better or worse than **Model 2**

if $P_1 - P_2 > 0$ or < 0

i.e. if $(C_0AC_0^1 - C_1B_0^1 - C_2V_0^1) - (C_0AC_0^2 - C_1B_0^2 - C_2V_0^2 - (IC_0)) > 0$ or < 0

$$\text{i.e. if} \begin{Bmatrix} C_2 < \text{ or } > C_{21}^{*} \text{ for } V_0^1 > V_0^2 \\ C_2 > \text{ or } < C_{21}^{*} \text{ for } V_0^1 < V_0^2 \end{Bmatrix}$$

Both are equally good if $C_2 = C_{21}^{*}$

$$\text{where } C_{21}^{*} = \frac{(C_0\left(AC_0^1 - AC_0^2\right) - C_1(B_0^1 - B_0^2) + IC_0)}{\left(V_0^1 - V_0^2\right)}$$

b) Comparing between **Model 2 and 3** reveals that **Model 2** is better or worse than **Model 3**

if $P_2 - P_3 > 0$ or < 0

i.e. if $(C_0AC_0^2 - C_1B_0^2 - C_2V_0^2 - (IC_0)) - (C_0AC_0^3 - C_1B_0^3 - C_2V_0^3 - 2*(IC_0)) > 0$ or < 0

$$\text{i.e. if} \begin{Bmatrix} C_2 < \text{ or } > C_{22}^{*} \text{ for } V_0^2 > V_0^3 \\ C_2 > \text{ or } < C_{22}^{*} \text{ for } V_0^2 < V_0^3 \end{Bmatrix}$$

Both are equally good if $C_2 = C_{22}^{*}$

$$\text{where } C_{22}^{*} = \frac{C_0\left(AC_0^2 - AC_0^3\right) - C_1\left(B_0^2 - B_0^3\right) + IC_0}{\left(V_0^2 - V_0^3\right)}$$

c) As far as the selection between Model 3 and 1 is concerned, one should adopt Model 3 in preference to Model 1

if $P_3 - P_1 > 0$ or < 0

i.e. if $(C_0AC_0^3 - C_1B_0^3 - C_2V_0^3 - 2*(IC_0)) - (C_0AC_0^1 - C_1B_0^1 - C_2V_0^1) > 0$ or < 0

$$\text{i.e. if} \begin{Bmatrix} C_2 < \text{ or } > C_{23}^{*}, \text{ for } V_0^3 > V_0^1 \\ C_2 > \text{ or } < C_{23}^{*}, \text{ for } V_0^3 < V_0^1 \end{Bmatrix}$$

Both are equally good if $C_2 = C_{23}^{*}$

$$\text{where } C_{23}^{*} = \frac{C_0(AC_0^3 - AC_0^1) - C_1(B_0^3 - B_0^1) - 2*\left(IC_0\right)}{(V_0^3 - V_0^1)}$$

V. CONCLUSION

Four reliability models have been developed to decide as to how many hot standby units should be there for a system working with two operative units. The decision may be taken by finding the difference between profits with regard to parameter of interest like revenue per unit up time, cost of installing a hot standby unit, cost per visit of the repairman or any other parameter which the user of such systems wishes to be considered. Cut-off points of some parameters have been obtained to reveal as to when and which model is more beneficial than the other. Cut-off points of some other parameters of interest may also be obtained to arrive at a decision of adopting one of the four discussed models.

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