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JOINING STRATEGIES IN MULTIPLE VACATION MACHINING SYSTEM WITH HETEROGENEOUS SERVERS

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Abstract: Markov machine repair model consisting of mixed spares under the supervision of two heterogeneous repairmen is investigated. Every time any component fails, it is quickly supplanted by a spare component if accessible. In the event when all spares are used up and the system operates with less than K functioning components then the failure of components happens in a degraded fashion. The distribution of failure time and repair time of the components are assumed to be exponential and FCFS discipline is used for repairing of failed components. The failed components may balk with constant probability as a result of impatience and may renege according to negative exponential distribution if the repairmen are busy. A reneged component can be retained in the queue by using some convincing ways to finish the repair process. After the repair, each component may rejoin the system as a feedback customer with some probability in case of imperfect repair. The repairmen switch to vacation state from busy state whenever there are no failed components in the system and repairmen switch back to the busy state as soon as any failed component arrives. The numerical technique named 'Successive Over Relaxation (SOR) Technique' is employed to acquire the system state probabilities at steady state which are then used to calculate the mean count of failed components in the system, throughput, carried load and other indices of the system. The numerical outcomes acquired are verified using the results generated by ANFIS.

Keywords: Balking; Mixed Spares; Vacation; Reneging; Retention; SOR; feedback

I. INTRODUCTION

The performance evaluation to the system of machines utilizing the approach of queueing theory is mostly done to solve several congestion circumstances in production systems and manufacturing systems; and to deal with the optimum control at many levels and issue of system design. The impatient behavior of failed machine components which upon arrival may or may not join the waiting line for repair depending on the count of failed machine components in the system, and of those which on entering and depart the queue without being repaired is studied by many researchers. Jain et al. [1] examined a machine repair problem with support of spares comprising of R permanent as well as r additional removable servers. Due to intolerance, the failed jobs may balk or renege if all servers are busy. Wang et al. [2] suggested a profit maximization for machine repair problem with R repairmen where switching of standbys fails according to a negative exponential distribution and failed machines may balk and renege on finding the repairmen busy. Maheshwari and Ali [3] investigated the system having functioning components together with the warm and cold spares comprising of the phenomenon of balking and reneging by using product type technique. Sharma [4] examined the M/M/2 machining

system with balking, reneging and vacation operating under N policy and also constructed the cost model for the same. The G/G/R machining system with balking and reneging using the approach of diffusion approximation is considered by Wang et al. [5]. They utilized Quasi- Newton method and the direct search method respectively, to find the optimal count of repairmen together with the optimal service rate so as to maximize the profit function.

The rest of the article is structured as follows. In Section 2, the assumptions and notations are outlined to explain the model. Chapman-Kolmogorov equations for the steady state using suitable rates of transition are built in Section 3. In Section 4, system characteristics are determined. The sensitivity analysis is performed to explore the impact of different parameters on system characteristics in Section 5. Finally, the study is concluded by underlining the noble features and future research directions in Section 6.

II. MODEL DESCRIPTION

In this section, we model a Markov queueing system by specifying the suitable rates of the transition of the birth-death process under study intended for the performance evaluation of a multi-component machining system. The system comprises of 'K' functioning components in addition to 'C' cold and 'W' warm spares (standbys) components. The life-times of the functioning components and spare components are distributed exponentially. The failure rate of functioning component is λ . The cold spares fail with rate '0' and the failure rate of a warm spare is $\varepsilon(<\lambda)$. The (n, K) policy is followed to run the system which states that if *n* \leq *no of functioning components* $\lt K$ where n $\lt K$, the system begin to run in short mode. The rate of failure of components in short mode rises to $\tilde{\lambda}$ (> λ) due to degradation of efficiency. The two heterogeneous repairmen are appointed at the repair desk for the repair of failed components. Only one failed component can be repaired at a time by each repairman. The probability that a failed component joins the queue when any of the two repairmen is on vacation or busy is denoted by *b* and balking probability is given by the complementary probability(1-b). After repair, each component may be sent back to rejoin the queue as a feedback component with probability α' if the repair is found to be imperfect else it leaves the system with satisfaction with complementary probability α . The state of the repairmen is denoted by 'j' as defined below:

- 0, *whenboththe servers are onvacation*
- 0, when both the servers are on vacation
1, when the first server is in busy state and the sec. *whenthe first server is inbusy state and the ond server is onvacation*
- 0, when both the
1, when the firs
2, when the sec
3, when both se. $=\begin{cases} 0 \\ 1 \\ 2 \\ 3 \end{cases}$ 0, when both the servers are on vacation
 $j =\begin{cases} 1, & \text{when the first server is in busy state and the second server is on vacation} \\ 2, & \text{when the second server is in busy state and the first server is on vacation} \end{cases}$ *when the first server is inbusy state*
*when the sec ond server is inbusy st
when both servers are inbusy state*

The repair times of failed components are exponentially distributed. The rate of repair of first (second) repairman changes from $\eta_1(\eta_2)$ to $\eta_1(\eta_2)$ and $\eta_1^{\dagger}(\eta_2^{\dagger})$ to $\eta_1^{\dagger}(\eta_2^{\dagger})$, whenever the count of failed components in the system reaches more than C and C+W, respectively. Also $\eta_1^* > \eta_1^* > \eta_1$ and $\eta_2^* > \eta_2^* > \eta_2^*$. The reneging of a failed component may happen according to an exponential distribution with rate ς . Each reneged component may be retained in the system with probability β ['] or else it leaves the system without being repaired with complementary probability β . The vacation times of both repairmen are supposed to be independent and identically distributed exponential random variables. $\psi_1(\psi_2)$ stands for the vacation rate by which first (second) repairman come back from vacation. The failed components are restored by following first come first repaired discipline. It is also assumed that the switchover times of components from failure to repair, from repair to spare and from spare to functioning states be negligible. It is to be noted that the cold spares are utilized before the warm spares to replace the failed components. After the restoration of failed components, they join the group of either spare components or functioning components depending upon whether the system is working in short or normal mode; the traits of these components are same as that of components of the functioning or spare set to which they join. The state-

dependent failure rate is given by
\n
$$
\lambda(m) = \begin{cases}\n(K\lambda + W\varepsilon), & m = 0 \\
(K\lambda + W\varepsilon)b, & 1 \le m < C \\
(K\lambda + (C + W - n))b, & C \le m < C + W \\
(K + C + W - n)\tilde{\lambda}b, & C + W \le m \le M - 1 \\
0 & otherwise\n\end{cases}
$$

Where, $M = K + C + W - n + 1$. The mean repair rate for the ith repairman is given by

$$
\eta_i(m) = \begin{cases} \alpha \eta_i, & 1 \le m < C \\ \alpha \eta_i, & C \le m < C+W \\ \alpha \eta_i, & C+W \le m \le M \end{cases}
$$

The reneging rate is given by $(m-1)$ $(m) = \begin{cases} 0 & 0 \le m \le 1 \\ (m-1)\beta\varsigma, & 2 \le m \le M \end{cases}$ $\gamma(m) = \begin{cases} 0 & 0 \le m \le 1 \\ (m-1)\beta\varsigma, & 2 \le m \le M \end{cases}.$ $0\leq m\leq 1$. The combination of

the repair rate and the reneging rate is given by:
\n
$$
\mu_i(m) = \eta_i(m) + \gamma(m) = \begin{cases}\n\alpha \eta_i, & m = 1 \\
\alpha \eta_i + (m - 1)\beta \zeta, & 2 \le m < C \\
\alpha \eta_i + (m - 1)\beta \zeta, & C \le m < C + W \\
\alpha \eta_i + (m - 1)\beta \zeta, & C + W \le m \le M \\
0, & \text{otherwise}\n\end{cases}
$$

The steady-state probabilities of the system state are defined as follows

: The probability that there are m $(0 \le m \le L)$ failed components in the system when both the repairmen are on vacation.

 \prod_m : The probability that there are m ($1 \le m \le L$) failed components in the system when the repairmen are in state j, $1 \le j \le 2$.

 $\prod_{m=1}^3$: The probability that there are m $(2 \le m \le L)$ failed components in the system when both the repairmen are busy in repairing the failed components.

III. THE ANALYSIS

The Chapman Kolmogorov equations are built in order to calculate the probabilities related with various states of the system by utilizing the suitable rates of fundamental birth-death process for $i=1$, 2 and $j=1,2$ as follows:

For $m = 0$; $(K\lambda + W\epsilon)\prod_{0}^{0} = \alpha \eta_1 \prod_{1}^{1} + \alpha \eta_2 \prod_{1}^{2}$ (1)

For $m=1$;

$$
((K\lambda + W\varepsilon) b + \psi_1 + \psi_2)\Pi_1^0 = (K\lambda + W\varepsilon)\Pi_0^0
$$
\n
$$
((K\lambda + W\varepsilon) b + \alpha \eta_i)\Pi_1^i = (\alpha \eta_i + \beta \varsigma)\Pi_2^i + (\alpha \eta_{j\neq i}^i + \beta \varsigma)\Pi_2^3 + \psi_i \Pi_1^0
$$
\n(3)

For $m = 2$;

$$
((K\lambda + W\varepsilon) b + \psi_1 + \psi_2)\Pi_2^0 = (K\lambda + W\varepsilon) b \Pi_1^0
$$

\n
$$
((K\lambda + W\varepsilon) b + (\alpha \eta_i + \beta \varsigma) + \psi_{j\alpha})\Pi_2^i = (\alpha \eta_i + 2\beta \varsigma) \Pi_3^i + (K\lambda + W\varepsilon) b \Pi_1^i + \psi_i \Pi_2^0
$$

\n(5)
\n
$$
((K\lambda + W\varepsilon) b + (\alpha (\eta_i + \eta_2) + 2\beta \varsigma))\Pi_2^3 = (\alpha (\eta_i + \eta_2) + 4\beta \varsigma) \Pi_3^3 + \psi_1 \Pi_2^2 + \psi_2 \Pi_2^1
$$

\n(6)

For $3 \le m \le C-2$;

T₂: The probability that there are m $((0 \le m \le 1)$ failed
components in the system when both the repairmen are on
weathed.
T₁: The probability that there are m $(1 \le m \le L)$ failed
components in the system when both the $((K\lambda + W\epsilon) b + \psi_1 + \psi_2)\prod_m^0 = (K\lambda + W\epsilon) b \prod_{m=1}^0$ (7) ((a) $\left((K\lambda + W\varepsilon) b + (\alpha \eta_i + (m-1)\beta \varsigma) + \psi_{j\neq i} \right) \prod_{m=1}^{i} = (\alpha \eta_i + m\beta \varsigma) \prod_{m=1}^{i} + (K\lambda + W\varepsilon) b \prod_{m=1}^{i} + \psi_i \prod_{m=1}^{0}$ ((K) (8) (8)
 $((K\lambda + W\varepsilon)b + (\alpha(\eta_1 + \eta_2) + 2(m-1)\beta\varsigma))] \Pi_m^3 = (\alpha(\eta_1 + \eta_2) + 2m\beta\varsigma) \Pi_{m+1}^3 + (K\lambda + W\varepsilon)b \Pi_{m-1}^3$ +*w* ϵ) D + $(\alpha \eta_i + (m-1)\beta \zeta)$ + $\psi_{j\neq i}$ | $\Gamma_{lm} = (\alpha \eta_i + m\beta \zeta) \Gamma_{lm+1} + (\kappa \lambda + w \zeta) D \Gamma_{lm-1} + \psi_i \Gamma_{lm}$

(8)

+ *W* ϵ) $b + (\alpha (\eta_i + \eta_2) + 2(m-1)\beta \zeta) \Pi_m^3 = (\alpha (\eta_i + \eta_2) + 2m\beta \zeta) \Pi_{m+1}^3 + (\kappa \lambda + W \zeta) b \Pi_{m-1}^3$

+ $\psi_i \$ $\sum_{i=1}^{n} \prod_{m=1}^{n} +\psi_2 \prod_{m=1}^{n}$ $R\lambda + W\epsilon D\theta + (\alpha \eta_1 + (m-1)\beta \zeta)^2 + \psi_{j\neq j} \prod_m - (\alpha \eta_1 + m\beta \zeta)^2 \prod_{m+1} + (\kappa \lambda + W\epsilon) D\Gamma \Gamma_{m-1} + \psi_i \Gamma_{m}$

(8)
 $R\lambda + W\epsilon D\theta + (\alpha (\eta_1 + \eta_2) + 2(m-1)\beta \zeta)) \prod_m^3 = (\alpha (\eta_1 + \eta_2) + 2m\beta \zeta) \prod_{m+1}^3 + (K\lambda + W\epsilon) D\Gamma_{m-1}^3$ $\left(\alpha\left(\eta_1+\eta_2\right)+2m\beta\varsigma\right)$
 $\psi_1\prod_m^2+\psi_2\prod_m^1$ (9)

For $m = C - 1$;

 $((K\lambda + W\epsilon) b + \psi_1 + \psi_2)\prod_m^0 = (K\lambda + W\epsilon) b \prod_{m=1}^0$ (10) ($\left((K\lambda + W\varepsilon)b + (\alpha\eta_i + (C-2)\beta\varsigma) + \psi_{j\neq i}\right)\prod_{c=1}^i = (\alpha\eta_i^{\prime} + (C-1)\beta\varsigma)\prod_{c}^i + (K\lambda + W\varepsilon)b\prod_{c=2}^i + \psi_i\prod_{c=1}^0$

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+ $w_{\mathcal{E}}$)b+ $\big(\alpha(\eta_i+\eta_2)+2(C-2)\beta_{\mathcal{S}}\big)\big)\Pi_{c-1}^3=\big(\alpha(\eta_i+\eta_2^*)+2(C-1)\beta_{\mathcal{S}}\big)\Pi_c^3+\big(K\lambda+W\mathcal{E}\big)b\,\Pi_{c-2}^3$ $\alpha(\eta_1^* + \eta_2^*) + 2(C-1)\beta_0$
 *C*₁ $\prod_{c=1}^2 + \psi_2 \prod_{c=1}^1$ *Chi* Colume - VIII, Issue - VII)
 K λ +*W* ε)*b* + $(\alpha(\eta_1 + \eta_2) + 2(C - 2)\beta_{\mathcal{S}})\Pi_{c-1}^3 = (\alpha(\eta_1^2 + \eta_2^2) + 2(C - 1)\beta_{\mathcal{S}})\Pi_c^3 + (K\lambda + W\varepsilon)b\Pi_{c-2}^3$

+ $\psi_1 \Pi_{c-1}^2 + \psi_2 \Pi_{c-1}^3$ $\left(\alpha \left(\eta_1^+ + \eta_2^+ \right) + 2(C-1) \beta \right)$
 $\psi_1 \prod_{c=1}^2 + \psi_2 \prod_{c=1}^1$ (12)

For $m = C$;

$$
((K\lambda + W\varepsilon) b + \psi_1 + \psi_2)\Pi_m^0 = (K\lambda + W\varepsilon) b \Pi_{m-1}^0
$$
\n
$$
((K\lambda + W\varepsilon) b + (\alpha \eta_i^2 + (C-1)\beta \varsigma) + \psi_{j\neq i})\Pi_c^i = (\alpha \eta_i^2 + C\beta \varsigma) \Pi_{c+1}^i + (K\lambda + W\varepsilon) b \Pi_{c-1}^i + \psi_i \Pi_c^0
$$
\n
$$
((K\lambda + W\varepsilon) b + (\alpha(\eta_i^2 + \eta_2^2) + 2(C-1)\beta \varsigma))\Pi_c^3 = (\alpha(\eta_i^2 + \eta_2^2) + 2C\beta \varsigma)\Pi_{c+1}^3 + (\kappa \lambda + W\varepsilon) b \Pi_{c-1}^3
$$
\n
$$
+ \psi_1 \Pi_c^2 + \psi_2 \Pi_c^i
$$
\n(15)

For $m = C + 1$;

$$
((K\lambda + (W-1)\varepsilon)b + \psi_1 + \psi_2)\Pi_{c+1}^{0} = (K\lambda + W\varepsilon)b\Pi_{c}^{0}
$$
\n(16)
\n
$$
((K\lambda + (W-1)\varepsilon)b + (\alpha\eta_i^{\prime} + C\beta\varsigma) + \psi_{j\neq i})\Pi_{c+1}^{i} = (\alpha\eta_i^{\prime} + (C+1)\beta\varsigma)\Pi_{c+2}^{i} + (K\lambda + W\varepsilon)b\Pi_{c}^{i} + \psi_i\Pi_{c+1}^{0}
$$
\n(17)
\n
$$
((K\lambda + (W-1)\varepsilon)b + (\alpha(\eta_i^{\prime} + \eta_i^{\prime}) + 2C\beta\varsigma))\Pi_{c+1}^{3} = (\alpha(\eta_i^{\prime} + \eta_i^{\prime}) + 2(C+1)\beta\varsigma)\Pi_{c+2}^{3} + (K\lambda + W\varepsilon)b\Pi_{c}^{3}
$$
\n
$$
+ \psi_1\Pi_{c+1}^{2} + \psi_2\Pi_{c+1}^{1}
$$
\n(18)

For $C+2 \le m \le C+W-2$;

(11) 0 0 1 2 ¹ *K C W m b K C W m b m m* (19) '' 0 1 1 1 1 *i i i j i m m i m i m K C W m b m K C W m b m* (20) '' 2 2 0 2 1 1 2 '' 2 2 1 1 1 *m m m m K C W m b m K C W m b m* (21)

For $m = C + W - 1$;

 $((K\lambda + (C+W-m)\varepsilon)b + \psi_1 + \psi_2)\Pi_m^0 = (K\lambda + (C+W-m+1)\varepsilon)b\Pi_m^0$ (22) $((K\lambda + (C+W-m)\epsilon)p + \psi_1 + \psi_2)\Pi_m - (K\lambda + (C+W-m+1)\epsilon)p\Pi_m$

(22)
 $((K\lambda + \epsilon)b + (\alpha\eta_i^* + (C+W-2)\beta\varsigma) + \psi_{j\epsilon i})\Pi'_{C+W-1} = (\alpha\eta_i^* + (C+W-1)\beta\varsigma)\Pi'_{C+W} + (K\lambda + 2\epsilon)b\Pi'_{C+W-2}$ $\left(\alpha\eta_i + C\right)$
 $W_i \prod_{c+W-1}^0$ + $(C+W-1)$
+W-1 +(C+w-m)&jp+ $\psi_1 + \psi_2$] $\prod_m = (\kappa \lambda + (\kappa + w - m + 1)\kappa) p \prod_m$

(22)

+ ε)b+ $(\alpha \eta_i^2 + (\zeta + w - 2)\beta \varsigma) + \psi_{j\alpha i}$] $\prod_{c+w-1}^i = (\alpha \eta_i^2 + (\zeta + w - 1)\beta \varsigma) \prod_{c+w}^i + (\kappa \lambda + 2\kappa) b \prod_{c+w-2}^i$ $K\lambda + (C+W-m)\epsilon\beta + \psi_1 + \psi_2\gamma\Gamma\Gamma_m = (K\lambda + (C+W-m+1)\epsilon)\beta\Gamma\Gamma_m$

(22)
 $K\lambda + \epsilon\beta + (\alpha\eta_i^2 + (C+W-2)\beta\varsigma) + \psi_{j\epsilon} \Gamma\Gamma_{c+w-1} = (\alpha\eta_i^2 + (C+W-1)\beta\varsigma)\Gamma_{c+w}^i + (\kappa\lambda + 2\epsilon)\beta\Gamma_{c+w-2}^i$ (23) $+\psi_{i} \prod_{c+w-1}^{\circ}$ (23)
($(K\lambda + \varepsilon)b + (\alpha(\eta_{1}^{'} + \eta_{2}^{'})) + 2(C+W-2)\beta_{5})\Big) \prod_{c+w-1}^{3} = (\alpha(\eta_{1}^{''} + \eta_{2}^{''}) + 2(C+W-2)\beta_{5})\prod_{c+w}^{3}$ (23)

= $(\alpha(\eta_1^* + \eta_2^*) + 2(C+W-2)\beta \zeta) \prod_{c+w}^3$

+ $(K\lambda + 2\varepsilon)b \prod_{c+w-2}^3 + \psi_1 \prod_{c+w-1}^2 + \psi_2 \prod_{c+w-1}^1$ (24)

For $m = C + W$;

 $\left(K\tilde{\lambda}b+\psi_1+\psi_2\right)\prod_{C+W}^{0}=\left(K\lambda+\varepsilon\right)b\prod_{C+W-1}^{0}$ (25) (25)
 $\begin{aligned} (25) \ (\bar{K} \bar{\lambda} b + (\alpha \eta_i^* + (C + W - 1) \beta \varsigma) + \psi_{j \neq i}] \Pi_{c+w}^i = (\alpha \eta_i^* + (C + W) \beta \varsigma) \Pi_{c+w+1}^i + (\bar{K} \lambda + \varepsilon) b \Pi_{c+w-1}^i + \psi_i \Pi_{c+w}^0 \end{aligned}$ (26)

(31)

COMPUSOFT, An international journal of advanced computer
 $\begin{aligned} &\text{COMPUSOFF, An international journal of advanced computer}: \\ &\int_{\tilde{\mathcal{T}}} + \eta_2^{\circ} \bigl) + 2 (C + W - 1) \beta_5 \bigr) \prod_{c+w}^3 = \bigl(\alpha \bigl(\eta_1^{\circ} + \eta_2^{\circ} \bigr) + 2 (C + W) \beta_5 \bigr) \prod_{c+w+1}^3 + (K \lambda + \varepsilon) b \prod_{c+w-1}^3 \\ &+ \psi_1 \$ $\alpha(\eta_1 + \eta_2) + 2(C +$
¹ $\prod_{c+W}^2 + \psi_2 \prod_{c+W}^1$ + η_2) + 2(C+W) μ
+w + ψ_2 \prod_{c+w}^{1} COMPUSOFT, An international journal of advanced computer 1
+ $(\alpha(\eta_1^*+\eta_2^*)+2(C+W-1)\beta_S))\prod_{c+w}^3 = (\alpha(\eta_1^*+\eta_2^*)+2(C+W)\beta_S)\prod_{c+w+1}^3 + (K\lambda+\varepsilon)b\prod_{c+w-1}^3$ = $(\alpha(\eta_1^* + \eta_2^*) + 2(C+W)\beta_{\varsigma})$
+ $\psi_1 \prod_{c+w}^2$ + $\psi_2 \prod_{c+w}^1$ *COMPUSOFT, An international journal of advanced compute*
 $K\bar{\lambda}b + (\alpha(\eta_{i}^{*}+\eta_{2}^{*})+2(C+W-1)\beta_{S}))\prod_{c+w}^{3} = (\alpha(\eta_{i}^{*}+\eta_{2}^{*})+2(C+W)\beta_{S})\prod_{c+w+1}^{3} + (K\lambda + \varepsilon)b\prod_{c+w-1}^{3}$ $\left(\alpha (\eta_1^* + \eta_2^*) + 2(C+W) \mu \right)$
 $\psi_1 \prod_{c+w}^2 + \psi_2 \prod_{c+w}^1$ (27)

For
$$
C+W+1 \le m \le M-1
$$
,
\n
$$
\left(\left(K + C + W - m \right) \tilde{\lambda} b + \psi_1 + \psi_2 \right) \prod_m^0 = \left(K + C + W - m + 1 \right) \tilde{\lambda} b \prod_{m=1}^0
$$
\n(28)
\n
$$
\left(\left(K + C + W - m \right) \tilde{\lambda} b + \left(\alpha \eta_i^{\dagger} + \left(m - 1 \right) \beta \zeta \right) + \psi_{j\alpha j} \right) \prod_m^i = \left(\alpha \eta_i^{\dagger} + m \beta \zeta \right) \prod_{n=1}^i + \left(K + C + W - m + 1 \right) \tilde{\lambda} b \prod_{n=1}^i
$$
\n(29)
\n
$$
\left(\left(K + C + W - m \right) \tilde{\lambda} b + \left(\alpha \left(\eta_i^{\dagger} + \eta_2^{\dagger} \right) + 2 \left(m - 1 \right) \beta \right) \right) \prod_m^3 = \left(K + C + W - m + 1 \right) \tilde{\lambda} b \prod_{n=1}^3 + \psi_1 \prod_m^2 + \psi_2 \prod_m^1
$$

(29)
 $=(K + C + W - m + 1) \tilde{\lambda} b \prod_{m=1}^{3} + \psi_1 \prod_m^2$
 $+ (\alpha (\eta_1^* + \eta_2^*) + 2m\beta \varsigma) \prod_{m=1}^{3}$ $C+W-m+1\big)\tilde{\lambda}b\prod$
 $\bar{\lambda}+\eta_{2}^{+}$ + 2m $\beta\varsigma\big)\prod_{m=1}^{3}$ (30)

For $m = M$;

 $(\psi_1 + \psi_2)\prod_M^0 = n\tilde{\lambda}b\prod_{M-1}^0$

For $C+W+1 \le m \le M-1$;

$$
\left(\left(\alpha \eta_i^+ + (M - 1) \beta \varsigma \right) + \psi_{j \neq i} \right) \prod_M^i = n \tilde{\lambda} b \prod_{M-1}^i + \psi_i \prod_M^0
$$
\n(32)\n
$$
\left(\alpha \left(\eta_1^* + \eta_2^* \right) + 2 \left(M - 1 \right) \beta \varsigma \right) \prod_M^3 = n \tilde{\lambda} b \prod_{M-1}^3 + \psi_1 \prod_M^2 + \psi_2 \prod_M^1
$$
\n(33)

[EM-(elg(xing)-2(xin x-3)(xin (elg(xing)-2(xin x-3)(xin (elg(xing)-2(xin x-3)(xin (xin x-7)(xin It is difficult to tackle the Chapman-Kolmogorov equations outlined for the continuous Markov chain of the model in consideration analytically since the recursive methodology involves unwieldy algebraic manipulation. So in such a case, the well-known numerical technique named 'Successive Over Relaxation (SOR) Technique' can be effectively employed to get the probabilities related with a large state space when the equations at steady state are already developed. An interesting fact about SOR technique is that it is the variant of Gauss–Seidel method wherein the rate of convergence is speeded up by choosing the apt relaxation parameter $\omega(1 \le \omega \le 1.25)$. Example $x_1x_2x_3x_4x_5x_6x_7x_2x_3x_1x_2x_3x_3x_4x_5x_6x_7x_2x_3x_1x_2x_3x_4x_4x_5x_5x_6x_6x_7x_7x_8x_8x_9x_1x_1x_2x_3x_4x_5x_6x_7x_7x_8x_8x_9x_1x_1x_2x_3x_4x_4x_5x_7x_7x_8x_8x_9x_9x_1x_1x_2x_3x_4x_4x_5x_7x_7x_8x_8x_9x_$

IV. SYSTEM CHARACTERISTICS

The main objective of obtaining probabilities in the preceding section is to evaluate various system characteristics such as mean count of failed components in the system, mean count of failed components in the queue, throughput, etc. to inspect the working of the model in consideration.

Mean count of failed components in the system is:
\n
$$
E[s] = \sum_{m=0}^{M} m \prod_{m}^{0} + \sum_{i=1}^{2} \sum_{m=1}^{M} m \prod_{m}^{i} + \sum_{m=2}^{M} m \prod_{m}^{3}
$$
\n(34)

Throughout is obtained using:
\n
$$
T = (\alpha \eta_1 \Pi_1^1 + \alpha \eta_2 \Pi_1^2) + \sum_{m=2}^{C-1} [\alpha \eta_1 \Pi_m^1 + \alpha \eta_2 \Pi_m^2 + (\alpha \eta_1 + \alpha \eta_2) \Pi_m^3]
$$

$$
+\sum_{m=C}^{C+W-1}\left[\alpha\eta_1^{'}\prod_m^{1}+\alpha\eta_2^{'}\prod_m^{2}+(\alpha\eta_1^{'}+\alpha\eta_2^{'})\prod_m^{3}\right]
$$

$$
+\sum_{m=C+W}^{M}\left[\alpha\eta_{1}^{*}\prod_{m}^{1}+\alpha\eta_{2}^{*}\prod_{m}^{2}+(\alpha\eta_{1}^{*}+\alpha\eta_{2}^{*})\prod_{m}^{3}\right]
$$
\n(35)

Probability that both repairmen are on vacation is given by $P[V] = \sum \prod_{i=1}^{n}$ $[V] = \sum_{m=0}^{M} \prod_{n=0}^{M}$ $P[V] = \sum_{m=0}^{N} \prod_{m=0}^{N}$

$$
(36)
$$

*Probability that only i*th (*i*=1, 2) repairman in a busy *state is obtained using* $P[B_i] = \sum_{m=1}^{M} \prod_{m=1}^{i}$ (37)

Probability that both repairmen are in the busy state

is:
$$
P[B] = \sum_{m=2}^{M} \prod_{m}^{3}
$$
 (38)

Average Balking Rate

Average Balking Rate
\n
$$
B.R. = (K\lambda + W\epsilon)\bar{b} \left[\sum_{j=0}^{2} \prod_{i=1}^{j} + \sum_{j=0}^{3} \sum_{m=2}^{C-1} \prod_{m}^{j} \right] + \sum_{j=0}^{3} \sum_{m=C}^{C+W-1} (K\lambda + (C+W-m)\epsilon)\bar{b} \prod_{m=0}^{j} + \sum_{m=C+W}^{3} \sum_{m=C+W}^{M-1} (K+C+W-m)\bar{\lambda}\bar{b} \prod_{m=0}^{j} \prod_{m=0}^{j} \tag{39}
$$

*Average Reneging Ra*te

$$
R.R. = \sum_{m=2}^{M} \left[(m-1) \beta \zeta \prod_{m=1}^{N} + (m-1) \beta \zeta \prod_{m=1}^{N} + 2(m-1) \beta \zeta \prod_{m=1}^{N} \right]
$$
(40)

Average Retention Rate

$$
\text{Average Retention Rate}
$$
\n
$$
\text{Ret} = \sum_{m=2}^{M} \left[(m-1)(1-\beta)\varsigma \prod_{m}^{1} + (m-1)(1-\beta)\varsigma \prod_{m}^{2} + 2(m-1)(1-\beta)\varsigma \prod_{m}^{3} \right]
$$
\n
$$
\tag{41}
$$

Loss Rate L R B R R R (42)

V. SENSITIVITY ANALYSIS

The software 'MATLAB' is utilized for analyzing the system numerically. We use the SOR technique to calculate the system indices of the M/M/2 Multiple Vacation Machining System with mixed standbys and impatience and then we compute the neuro-fuzzy results by utilizing the neuro-fuzzy tool in the software 'MATLAB'. For the purpose of illustration, we fix the parameters as $K=6$, C=3,W=2, n=3, $\lambda = 0.2$, $\lambda_d = 0.9$, $\varepsilon = 0.1$, $\eta_1 = \eta_2 = 0.5$, $\eta_1 = \eta_1, \ \eta_1 = 2\eta_1, \ \eta_1 = 3\eta_1, \ \eta_2 = \eta_2, \ \eta_2 = 2\eta_2, \ \eta_2 = 3\eta_2,$ $\alpha = 0.5$, $\beta = 0.5$, $\psi_1 = 0.3$, $\psi_2 = 0.1$, b=0.8, $\zeta = 0.02$

The outcomes gotten by utilizing SOR technique for the mean count of failed components in the system and throughput are plotted by utilizing smooth and dashed lines in Fig. 2(a–b) and Fig. 3(a-b) respectively alongside the ANFIS outcomes shown using tick marks. In Fig 2(a-b), we have displayed the numerical results for mean count of failed components in the system E[s] by varying λ for different values of (a) b and (b) ζ . We can clearly see that with increase in arrival rate λ , E[s] also increases. Also, the

results for throughput 'T' that are obtained numerically are shown in Fig. 3(a-b) by varying failure rate λ for different values of (a) b and (b) ζ where throughput T rises with rise in λ . It is obvious from these Figs that the outcomes acquired by both SOR and ANFIS are quite close and from the patterns of the outcomes showed in these Figs, it may be inferred that the outcomes gotten by ANFIS are at standard with the SOR outcomes.

Overall it is concluded that the mean count of failed components in the system E[s] increases (decreases) with the increment in the $\mathfrak{b}(\mathcal{G})$; this pattern is same what we expect in real time systems. The throughput of the system T increases (decreases) with the rise in value of b(ς)which also coincides with the real world situation. The results obtained from the ANFIS approach are quite accurate and are at par with the results acquired from SOR approach.

VI. CONCLUSION

This article analyzes the multi-component machining system by incorporating features of mixed spares, two repairmen, vacation, balking, feedback, reneging and retention that makes our model more versatile from an application viewpoint. Our investigation may provide some administrative bits of knowledge with respect to the upkeep for machining systems, like, manufacturing processes, transportation frameworks, and so on. The visualization of the impact of various parameters on the system indices is done by carrying out sensitivity analysis. The total cost of the system is also obtained. The proposed model can help the system engineers for enhancing the availability and reliability of the machining framework by giving suitable spare backing and restoration facility. Moreover, to extend the current model, the features like N policy, F-policy and unreliable servers can be included. The reasonable elements of bulk failure can also be incorporated, but the assessment of system indices, in this case, appears to be complicated.

VII. REFERENCE

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Fig. 1. Membership function for λ

Figure 2: E(s) vs. λ for different values of (a) b and (b) ζ

Fig. 3: T vs. for different values of (a) b and (b) $\boldsymbol{\zeta}$.