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OPTIMAL SCHOOL BUS ROUTES USING MIXED–INTEGER PROGRAMMING FOR LONG-TERM SCHOOL BUS SYSTEM

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Abstract: In this paper, we apply the mixed–integer programming model for solving the modified school bus routing problem under the given list of bus stops and the distances between each pair of bus stops without the number of students in each bus stop. From this model, the obtained schedule can be used for constructing the registration bus system in the long-term, resulting in a decreasing of 55.04% in the total distance of all bus routes as compared to the current routes of a secondary school located in the urban fringe of Thailand.

Keywords: Mixed integer programming; travelling salesman problem; school bus route optimization

I. INTRODUCTION

The vehicle routing problem (VRP) is the problem concerning the combinatorial optimization and the integer program determining the optimal routes for multiple vehicles visiting a set of locations. This problem first appeared in the publication of the article on “the truck dispatching problem” of Dantzig and Ramser [1] in 1959. The goal of the truck dispatching problem (TDP) is to seek the optimal routes of gasoline delivery trucks between a large terminal and a numerous number of service stations which are supplied by the terminal. This problem is one of the generalizations of the classical problem like the travelling salesman problem (TSP). The TDP and the suggested algorithm for finding the petrol deliveries in the TDP of Dantzig and Ramser [1] opened an avenue for further development of several algorithms and numerous heuristics for solving various problems of the VRP. The examples of techniques for solving the VRP are as follows:

- branch–and–bound algorithm of Christofides and Eilon [2];
- dynamic programming formulation of Eilon, Watson-Gandy, and Christofides [3];
- vehicle flow formulation of Laporte and Nobert [4];
- set partitioning formulation of Balinski and Quandt [5];
- the savings algorithm of Clarke and Wright [6];
- sweep algorithm of Gillett and Miller [7];
- cluster-first, route-second heuristic of Fisher and Jaikumar [8];
- local search such as tabu search, Genetic Algorithm [14], simulated annealing[15], variable neighborhood search, very large neighborhood search, adaptive large neighborhood search etc.

Nowadays, the VRP is one of the hearts of distribution management because a lot of companies and organizations in each day are engaged in the delivery and collection of products or people.

The school bus routing problem (SBRP) is the VRP which has received interest for over 50 years since the appearance of the first article of Newton and Thomas [9] in 1969. The main aim of the SBRP seeks to design the efficient schedule for a fleet of school buses that transport children from several bus stops to schools concerning various conditions via the transportation system such as the capacity of a bus, and the time window of a school, etc. The following conditions are some common specific characteristics of the SBRP.

1. The routes of each bus in the SBRP are the paths in the SBRP.
2. The total number of students in each bus cannot exceed the capacity of the bus.
3. The distance or time of each path is limited by the appropriate time.

Currently, the school bus system is served in numerous schools in the real–life. In this system, the bus routes and bus stops are determined by the school. Then students choose available bus stops from the given bus routes. However, some schools lack the effective planning of the bus routes causing the high cost of the school, the time wasted in the travel to school, etc.

Based on the above-described problem, the number of students in each bus stop is known after the registration is completed. The main aim of this work is to apply the mixed–integer programming for solving this problem, which is similar to the SBRP, but the second characteristic of the SBRP is not considered. The obtained results can be constructed the bus routes for the school bus system in the long–term.

The remainder of this research is arranged as follows. The problem description in this research and the mixed–integer programming formulation are described in Section 2. In Section 3, the computational solution of the proposed model is given, and the comparison between the results from the model with the current routes of a secondary school located in the urban zone of Thailand is discussed.

II. PROBLEM DESCRIPTION AND MIXED–INTEGER PROGRAMMING FORMULATION

The problem in this research is considered on a complete graph $G=(V,A)$, where $V=I\cup\{0\}$ is the set of all nodes such that 0 denotes a school and I is the set of intermediate bus stops. The distance between each two nodes is given by a matrix $D=\left[d_{ij}\right]$, where d_{ij} represents the distance from node i to node j. The problem consists of determining N nodes disjoint paths connected to a school such that the total distance of each path does not exceed the given distance T. Next, a dummy node d is introduced as the initial nodes of all paths. From this setting, the problem become to determining N nodes disjoint paths between two nodes on a graph $G'=(V',A')$ which is an expanded graph of $G=(V,A)$, where $V'=V\cup\{d\}$. In the expanded graph, the new distance between each two nodes in V' is given by a matrix $D'=\left[d'_{ij}\right]$, where,

$$d'_{ij} = \begin{cases} 0 & \text{if } i=d \text{ and } j\in I \\ M & \text{if } i=d \text{ and } j=0 \\ d_{ij} & \text{otherwise.} \end{cases}$$

Before formulating the mixed–integer programming model, the decision variables are defined as follows:

$$x_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is traversed in the solution} \\ 0 & \text{otherwise,} \end{cases}$$

v_i denotes the distance that the bus travelled up until node $i\in V'$.

We now present the mixed–integer programming model representing the focuses problem to determine the optimal planning of bus routes.

Minimize

$$\sum_{i\in V'}\sum_{j\in V'} d_{ij}x_{ij} \tag{1}$$

subject to

$$\sum_{i\in I} x_{i0} = k \tag{2}$$

$$\sum_{i\in I} x_{di} = k \tag{3}$$

$$\sum_{j\in I\cup\{0\}} x_{ij} = 1 \qquad \forall i\in I \tag{4}$$

$$\sum_{i\in I\cup\{d\}} x_{ij} = 1 \qquad \forall i\in I \tag{5}$$

$$v_i - v_j + \left(T-d_{j0}+d_{ij}\right)x_{ij} + \left(T-d_{j0}-d_{ji}\right)x_{ji} \le T-d_{j0} \qquad \forall i,j\in I, i\neq j \tag{6}$$

$$v_i - d_{i0}x_{i0} \ge 0 \qquad \forall i\in I \tag{7}$$

$$v_i - d_{i0}x_{i0} + Tx_{i0} \le T \qquad \forall i\in I \tag{8}$$

$$x_{ij}\in\{0,1\} \qquad \forall i,j\in I' \tag{9}$$

$$v_i \ge 0 \qquad \forall i\in I. \tag{10}$$

In this model, the objective function indicates the total distance of all bus routes. Constraints (2) and (3) allow at k buses to transport students. Constraints (4) and (5) are the degree constraints ensuring that each intermediate node to be visited exactly once. Constraints (6) are the subtour elimination constraints, whereas (7) and (8) are specifically derived for the problem (see more details in the articles of Desrochers and Laporte [10], Naddef [11], Kara and Bektas [12], Bektas and Elmastas [13]). Constraints (9) and (10) correspond to the variable definition.

III. THE SOLUTION OF THE MODEL WITH A CASE STUDY

In this section, the mixed–integer programming model (1)–(10) in Section 2 was applied to a real case in a secondary school located in the Pathumthani, Thailand in 2018, and the results are compared with the solution provided by the school. In the case study, the transportation of the students is managed by the school bus system, which has 26 buses with the capacity of 70 students, and there are 1,415 students to be picked up from 107 bus stops of the school. The total distance of each bus is in Fig. 1.

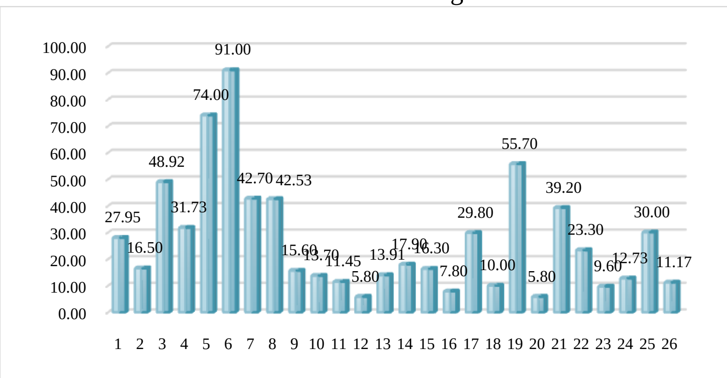

Figure 1: The total distance of each bus in a secondary school located in the Pathumthani, Thailand in 2018.

Fig. 1 presents the total distance of 26 buses of a secondary school located in the Pathumthani, Thailand in 2019. From this figure, it is clear that the longest bus route is 91 kilometre, the shortest bus route is 5.8 kilometre, and the total distance of all bus routes is 705.09 kilometre. Furthermore, the average distance of all bus routes is 27.12 kilometre.

The results of the mixed–integer programming model are obtained from the computation via CPLEX 12.6.1, and are shown in Fig. 2.

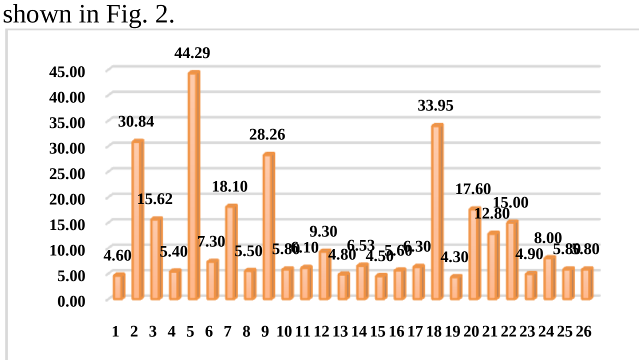

Figure 2: The total distance of each bus from the model

Fig. 2 presents the total distance of 26 buses which are obtained from the mixed–integer programming model. From this figure, the longest bus route is 44.29 kilometre, the shortest bus route is 4.50 kilometre, and the total distance of all bus routes is 316.99 kilometre. Accordingly, the total distance of all bus routes has a decrement 55.04% and so the performance of the system is increasing.

One of the most practical decisions is to plan how many buses used in the system. Table 1 presents the results from the mixed–integer programming model (1)–(10) in Section 2 with the range of the number of school buses is 22–30 such that the minimum number in this range is the possible number of school buses covering 1,415 students while the number in the range 27–30 is prepared in order to make the school bus system in the future if the number of students is increasing.

Table 1. Results from the model (1) – (10) in the case of $N=22$ to $N=30$.

N	Max route (km)	Min route (km)	Total distance (km)	Difference (%)
22	46.193	4.3	292.54	58.51
23	47.093	4.3	298.59	57.65
24	46.193	4.3	304.69	56.79
25	47.393	4.3	310.79	55.92
26	44.293	4.3	316.99	55.04
27	44.293	4.3	323.26	54.15
28	44.293	4.3	329.56	53.26
29	44.293	4.3	335.96	52.35
30	44.293	4.3	342.56	51.42

The second column of Table 1 represents the longest distance bus route, whereas the third column is the shortest distance bus route. The total distance of all bus routes is given in the fourth column of Table 1. The last column is the percentage of the difference between the total distance of all bus routes with a total distance in the fourth column. From the view of the total distance of all bus routes, the effectiveness in the school bus system varies on the number of buses such that the total distance of all bus routes is increasing whenever the number of buses is increasing (see Fig. 3).

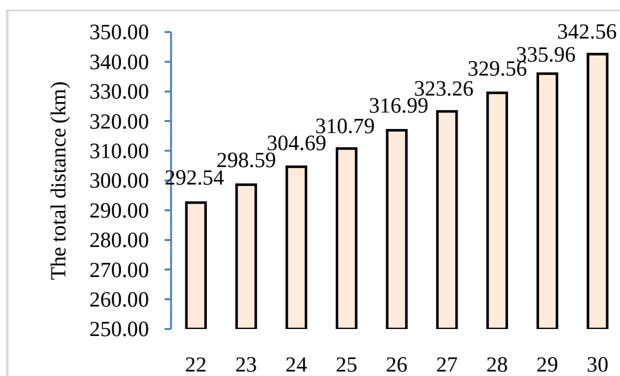

Figure 3: The total distance in the case of $N=22$ to $N=30$.

IV. CONCLUSION

In this paper, a real life school bus routing planning model based on a mixed–integer programming for a modified school bus routing problem was formulated. One of the advantages of this model is to use in the long–term for the school bus system since this model does not vary on the number of students in each bus stops. However, the total of students in each bus route is limited by the bus capacity. Furthermore, the model was applied in the real data from the secondary school located in the urban fringe of Thailand. The experimental results with the formulated model shown that it is able to obtain better solutions than those obtained by the traditional operation planning in the case of 26 buses such that the total distance of all bus routes is decreasing of 55.04%.

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