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THE EFFECT OF MEASUREMENT ERRORS ON THE DOUBLE SAMPLING $\bar{X}$ CHART

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Abstract: The purpose of this study is to investigate the performance of the double sampling (DS) Xbarchart when measurement errors exist. The effects of measurement error ratio,parameter of the linearly covariate error model and multiple measurements on the performance of the DS Xbar chart are evaluated. The numerical results show that the performance of the DS Xbar chart is affected by the presence of measurement errors. An example is presented to illustrate the application of the DS Xbar chart under measurement errors.

Keywords: average run length, double sampling, linearly covariate error model, measurement errors, $\bar{X}$ chart

I. INTRODUCTION

When designing a control chart, it is usually assumed that measurement errors do not exist in the process. However, measurement errors must be considered in practice since the performance of control charts is known to deteriorate in the presence of measurement errors.

Concerning the model for measurement errors, [1] used a linearly covariate measurement error model to study the effect of measurement errors on the performance of the $\bar{X}$ and S^2 charts. Recently, this model has been widely used by researchers (see, for example [2-10]) to evaluate the performance of various control charts with measurement errors.

In this study, the linearly covariate error model proposed by [1] is combined with the double sampling (DS) scheme of the $\bar{X}$ chart. The effect of measurement error ratio, the effect of parameter in the linearly covariate error model and the effect of multiple measurements on the performance of the DS $\bar{X}$ chart are evaluated.

II. LINEARLY COVARIATE ERROR MODEL

It is assumed that Y_{ij} is the quality characteristic to be monitored in the process and samples of size n_S are taken from the process at sampling point i = 1, 2, 3,… and j = 1, 2, 3, …, n_S for $S \in \{1, 2\}$, where n_1 is the first sample size and n_2 is the second sample size. It is also assumed that Y_{ij} follows a normal distribution with known in-control process mean μ_0 and process standard deviation σ_0, The process mean shift is $\delta = (\mu_0 - \mu)/\sigma_0$ when the process is out-of-control, where μ is the out-of-control process mean. Note that $\delta = 0$ when the process is in-control, while $\delta \neq 0$ when the process is out-of-control.

It has been suggested by many researchers to take multiple measurements per item in each sample, in order to compensate for the effect of measurement errors. It is assumed that Y_{ij} is not directly observable and Y_{ij} is estimated by using X_{ijk}, where k is the number of measurements for each item, for $k = 1, 2, \ldots, m$. Here, X_{ijk} is the true value of the quality characteristic and Y_{ij} is the observed value. For the linearly covariate error model in [1], it is assumed that the relation between X_{ijk} and Y_{ij} is as follows:

$$X_{ijk} = A + BY_{ij} + \varepsilon_{ijk}\ , \tag{1}$$

where A and B are known constants. Here, ε_{ijk} is the random measurement error term, which is normally distributed with mean 0 and standard deviation σ_ε. The mean for each sample is

$$\bar{X}_{iS} = \frac{1}{mn_S}\sum_{j=1}^{n_S}\sum_{k=1}^{m} X_{ijk}\ , \tag{2}$$

where n_1 is the sample size at the first stage of the DS scheme, while $(n_1 + n_2)$is the sample size at the second stage of the DS scheme. Substituting Equation (1) into Equation (2) gives

$$\bar{X}_{iS} = A + \frac{1}{n_S}\left(B\sum_{j=1}^{n_S} Y_{ij} + \frac{1}{m}\sum_{j=1}^{n_S}\sum_{k=1}^{m}\varepsilon_{ijk}\right). \tag{3}$$

The sample mean at each sampling stage of the DS scheme is

$$\bar{X}_i = \begin{cases} \bar{X}_{i1} & \text{at the first stage} \\ \dfrac{n_1\bar{X}_{i1} + n_2\bar{X}_{i2}}{n_1+n_2} & \text{at the second stage} \end{cases}, \tag{4}$$

where the mean and the variance of $\bar{X}_i$ [1] can be computed as

$$E\left(\bar{X}_i\right) = A + B\mu \tag{5}$$

and

$$V\left(\bar{X}_i\right) = \frac{1}{n}\left(B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{m}\right), \tag{6}$$

respectively, $n = n_1$ at the first stage of the DS scheme and $n = n_1 + n_2$ at the second stage of the DS scheme.

III. Double Sampling $\bar{X}$ Chart with Measurement Errors

The DS $\bar{X}$ chart was introduced by [11] under the assumption that the value of the quality characteristic is measured without errors. This study investigates the effect of measurement errors on the DS $\bar{X}$ chart by assuming the same model of measurement errors as in [1]. Based on Equations (5) and (6), the limits of the DS $\bar{X}$ chart in the presence of measurement errors ($\delta = 0$) are given as

$$\text{LCL}_1 = A + B\mu_0 - k_1\sqrt{\frac{1}{n_1}\left(B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{m}\right)}, \tag{7}$$

$$\text{LWL} = A + B\mu_0 - w\sqrt{\frac{1}{n_1}\left(B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{m}\right)}, \tag{8}$$

$$\text{UWL} = A + B\mu_0 + w\sqrt{\frac{1}{n_1}\left(B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{m}\right)}, \tag{9}$$

$$\text{UCL}_1 = A + B\mu_0 + k_1\sqrt{\frac{1}{n_1}\left(B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{m}\right)}, \tag{10}$$

$$\text{LCL}_2 = A + B\mu_0 - k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{m}\right)} \tag{11}$$

and

$$\text{UCL}_2 = A + B\mu_0 + k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{m}\right)}, \tag{12}$$

where LCL_1, LWL, UWL and UCL_1 are the lower control limit, the lower warning limit, the upper warning limit, the upper control limit at the first stage of the DS scheme, while LCL_2 and UCL_2 are the lower control limit and the upper control limit at the second stage of the DS scheme. Note that $B = 1$ and $\sigma_\varepsilon = 0$ if without measurement errors.

Since the sample size can be either n_1 or $(n_1 + n_2)$ depending on the position of the sampling point at the first stage of the DS scheme, it is difficult to directly monitor $\bar{X}_i$ since its distribution is different between using the sample sizes of n_1 and $(n_1 + n_2)$. As such, it is better to monitor its standardized value which is defined as

$$Z_i = \frac{\bar{X}_i - (A + B\mu_0)}{\sqrt{\frac{1}{n}\left(B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{m}\right)}}. \tag{13}$$

The limits of the DS $\bar{X}$ chart can be simplified to $\text{LCL}_1 = -k_1$, $\text{LWL} = -w$, $\text{UWL} = w$, $\text{UCL}_1 = k_1$, $\text{LCL}_2 = -k_2$, and $\text{UCL}_2 = k_2$ (see Figure 1). Consequently, the DS $\bar{X}$ chart has the following in-control regions: $I_1 = [-w, w]$; $I_2 = [-k_1, -w] \cup [w, k_1]$; and $I_3 = [-k_2, k_2]$, with the following out-of-control regions: $I_4 = (-\infty, -k_1) \cup (k_1, \infty)$ and $I_5 = (-\infty, -k_2) \cup (k_2, \infty)$.

Figure 1 is the graphical presentation of the DS $\bar{X}$ chart, where the chart can be constructed with two scales: the left scale is used to plot the sample at the first stage of the DS scheme with limits of $-k_1$, $-w$, w and k_1; while the right scale is used to plot the sample at the second stage of the DS scheme with limits of $-k_2$ and k_2. Two scales are adopted so that a single chart can be used, instead of using two different charts for the first and second stages of the DS scheme.

The operational procedure of the DS $\bar{X}$ chart is as follows:

Step 1. Take a sample of size n_1, then calculate $\bar{X}_i$ and Z_i using Equations (4) and (13), respectively, in which this is the first stage of the DS scheme. Based on the left scale of the DS $\bar{X}$ chart, there are three possibilities:

- If $Z_i \in I_1$, return to Step 1.
- If $Z_i \in I_2$, then go to the next step.
- If $Z_i \in I_4$, then go to Step 3.

Step 2. Take an additional sample of size n_2, then calculate $\bar{X}_i$ and Z_i using Eqs. (4) and (13), respectively, in which this is the second stage of the DS scheme. Based on the right scale of the DS $\bar{X}$ chart, there are two possibilities:

- If $Z_i \in I_3$, return to Step 1.
- If $Z_i \in I_5$, then go to the next step.

Step 3. An out-of-control signal is triggered and the corrective action(s) will be taken to remove the assignable cause(s) to bring the process back to the in-control state. Then, return to Step 1.

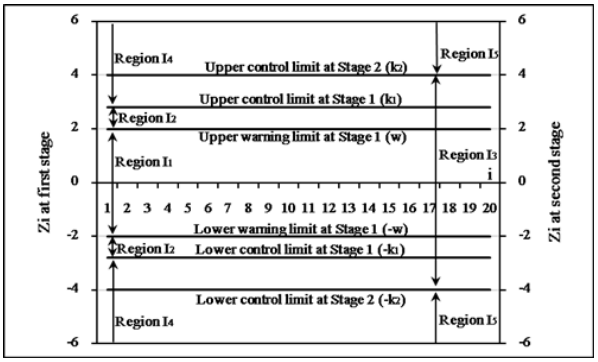

Figure 1: Graphical presentation of the DS $\bar{X}$ chart

IV. Performance Measure

Let p_a be the probability that a sample is plotted within the limits of the DS $\bar{X}$ chart under measurement errors, then $p_a = p_{a1} + p_{a2}$, where p_{a1} is the probability that a sample is plotted within Region I_1 at the first stage of the DS scheme, while p_{a2} is the probability that a sample is plotted within Region I_2 at the first stage of the DS scheme and then the sample is plotted within Region I_3 at the second stages of the DS scheme. The average run length of the DS $\bar{X}$ chart is given as

$$\mathrm{ARL} = \frac{1}{1-p_a}, \tag{14}$$

where p_{a1} and p_{a2} are determined as follows:

$$p_{a1} = \Phi\left(w + \frac{B\delta\sqrt{n_1}}{\sqrt{B^2+\gamma^2/m}}\right) - \Phi\left(-w + \frac{B\delta\sqrt{n_1}}{\sqrt{B^2+\gamma^2/m}}\right) \tag{15}$$

and

$$p_{a2} = \int_{I_2}\left(\Phi\left(k_2\sqrt{\frac{n_1+n_2}{n_2}} + \frac{B\delta(n_1+n_2)}{\sqrt{n_2(B^2+\gamma^2/m)}} - z\sqrt{\frac{n_1}{n_2}}\right) - \Phi\left(-k_2\sqrt{\frac{n_1+n_2}{n_2}} + \frac{B\delta(n_1+n_2)}{\sqrt{n_2(B^2+\gamma^2/m)}} - z\sqrt{\frac{n_1}{n_2}}\right)\right)\phi(z)dz, \tag{16}$$

where $\gamma^2 = \frac{\sigma_\varepsilon^2}{\sigma_0^2}$ is the measurement error ratio, $I_2 \in [-k_1, -w] \cup [w, k_1]$; $\Phi(\cdot)$ is the cumulative distribution function of the standardized normal random variable and $\phi(z)$ is the probability density function of the standardized normal random variable. The details of the derivation p_{a2} are given in Appendix.

It can be noticed in Equations (15) and (16) that p_{a1} and p_{a2} do not depend on the constant A of the linearly covariate error model. Thus, the value of A has no influence on the ARL performance of the DS $\bar{X}$ chart with measurement errors. The in-control average sample size of the DS $\bar{X}$ chart is calculated as

$$\bar{n} = n_1 + n_2\{\Phi(k_1) - \Phi(w)] + [\Phi(-w) - \Phi(-k_1)\}. \tag{17}$$

V. Result and Finding

In this section, the effect of measurement errors, the effect of constant B and the effect of multiple measurements on the performance of the DS $\bar{X}$ chart will be investigated. Let ARL_0 and ARL_1 be the in-control and out-of-control ARLs, respectively. For a fair comparison, all the control charts in Tables I-III have the same in-control performance (i.e. $\mathrm{ARL}_0 = 370$ and $\bar{n} = 5$ or 10) and the control chart with the smaller ARL_1 has a better performance. The design parameters of the DS $\bar{X}$ chart in Tables I-IIIare directly obtained from [12], where the DS $\bar{X}$ chart is optimized at $\delta_{\mathrm{opt}} \in \{0.5, 1.0\}$.

Table I compares the values of ARL_1 in terms of different measurement error ratios $\gamma^2 \in \{0, 0.1, 0.5, 1\}$ when $m = 1$ and $B = 1$. From the results in Table I, we can see that the ARL_1 values without considering the measurement errors (when $\gamma^2 = 0$) are smaller compared to those with measurement errors (when $\gamma^2 = 0.1, 0.5, 1$). As expected, the DS $\bar{X}$ chart performs better (i.e. smaller ARL_1) with the smaller measurement error ratio γ^2.

In Table II, the values of ARL_1 are compared in terms of different values of $B \in \{0.5, 1, 1.5, 2\}$ when $m = 1$ and $\gamma^2 = 1$. It can be noticed that the negative effect of measurement errors on the performance of the DS $\bar{X}$ chart decreases (i.e. smaller value of ARL_1) with the increase of the value in B.

Table III shows the comparison of ARL_1 for different values of $m \in \{1, 2, 3, 4\}$ when $B = 1$ and $\gamma^2 = 1$. It can be seen that the negative effect of measurement errors on the performance of the DS $\bar{X}$ chart decreases as the value of m increases.

From the results in Tables I-III, it can be concluded that the measurement errors have a negative effect on the performance of the DS $\bar{X}$ chart. This negative effect can be reduced by increasing the values of B and m.

TABLE I. ARL_1 FOR $\mathrm{ARL}_0 = 370$, $M = 1$, $B = 1$ FOR DIFFERENT γ^2

$n_1 = 4$, $n_2 = 10$, $w = 1.63837$, $k_1 = 3.20638$ and $k_2 = 3.003$ ($\bar{n} = 5$, $\delta_{\mathrm{opt}} = 0.5$ or 1.0)				
δ	$\gamma^2 = 0$	$\gamma^2 = 0.1$	$\gamma^2 = 0.5$	$\gamma^2 = 1$
0.1	247.82	255.90	279.76	298.46
0.5	12.02	13.88	21.86	32.44
1	1.77	1.95	2.82	4.20
1.5	1.10	1.13	1.30	1.59
2	1.01	1.02	1.06	1.14
$n_1 = 8$, $n_2 = 20$, $w = 1.63837$, $k_1 = 3.20638$ and $k_2 = 3.003$ ($\bar{n} = 10$, $\delta_{\mathrm{opt}} = 0.5$)				
δ	$\gamma^2 = 0$	$\gamma^2 = 0.1$	$\gamma^2 = 0.5$	$\gamma^2 = 1$

0.1	181.11	190.85	221.57	247.82
0.5	4.20	4.83	7.74	12.02
1	1.14	1.18	1.40	1.77
1.5	1.00	1.01	1.04	1.10
2	1.00	1.00	1.00	1.01

n_1 = 8, n_2 = 16, w = 1.52867, k_1 = 3.20605 and k_2 = 3.064
($\bar{n}$ = 10, δ_{opt} = 1.0)

δ	$\gamma^2 = 0$	$\gamma^2 = 0.1$	$\gamma^2 = 0.5$	$\gamma^2 = 1$
0.1	190.74	200.35	230.36	255.67
0.5	4.74	5.49	8.89	13.85
1	1.13	1.17	1.42	1.86
1.5	1.00	1.01	1.03	1.09
2	1.00	1.00	1.00	1.01

TABLE II. ARL_1 FOR ARL_0 = 370, M = 1, γ^2 = 1 FOR DIFFERENT B

n_1 = 4, n_2 = 10, w = 1.63837, k_1 = 3.20638 and k_2 = 3.003
($\bar{n}$ = 5, δ_{opt} = 0.5 or 1.0)

δ	B = 0.5	B = 1	B = 1.5	B = 2
0.1	338.04	298.46	277.06	266.19
0.5	91.47	32.44	20.72	16.79
1	16.79	4.20	2.69	2.25
1.5	4.91	1.59	1.27	1.19
2	2.25	1.14	1.05	1.03

n_1 = 8, n_2 = 20, w = 1.63837, k_1 = 3.20638 and k_2 = 3.003
($\bar{n}$ = 10, δ_{opt} = 0.5)

δ	B = 0.5	B = 1	B = 1.5	B = 2
0.1	310.73	247.82	217.94	203.74
0.5	43.14	12.02	7.30	5.86
1	5.86	1.77	1.36	1.25
1.5	1.97	1.10	1.03	1.02
2	1.25	1.01	1.00	1.00

n_1 = 8, n_2 = 16, w = 1.52867, k_1 = 3.20605 and k_2 = 3.064
($\bar{n}$ = 10, δ_{opt} = 1.0)

δ	B = 0.5	B = 1	B = 1.5	B = 2
0.1	315.17	255.67	226.84	213.00
0.5	48.51	13.85	8.39	6.69
1	6.69	1.86	1.38	1.25
1.5	2.10	1.09	1.02	1.01
2	1.25	1.01	1.00	1.00

TABLE III. ARL_1 FOR ARL_0 = 370, B = 1, γ^2 = 1 FOR DIFFERENT M

n_1 = 4, n_2 = 10, w = 1.63837, k_1 = 3.20638 and k_2 = 3.003
($\bar{n}$ = 5, δ_{opt} = 0.5 or 1.0)

δ	m = 1	m = 2	m = 3	m = 4
0.1	298.46	279.76	271.15	266.19
0.5	32.44	21.86	18.45	16.79
1	4.20	2.82	2.43	2.25
1.5	1.59	1.30	1.22	1.19
2	1.14	1.06	1.04	1.03

n_1 = 8, n_2 = 20, w = 1.63837, k_1 = 3.20638 and k_2 = 3.003
($\bar{n}$ = 10, δ_{opt} = 0.5)

δ	m = 1	m = 2	m = 3	m = 4
0.1	247.82	221.57	210.14	203.74
0.5	12.02	7.74	6.46	5.86
1	1.77	1.40	1.30	1.25
1.5	1.10	1.04	1.02	1.02
2	1.01	1.00	1.00	1.00

n_1 = 8, n_2 = 16, w = 1.52867, k_1 = 3.20605 and k_2 = 3.064
($\bar{n}$ = 10, δ_{opt} = 1.0)

δ	m = 1	m = 2	m = 3	m = 4
0.1	255.67	230.36	219.24	213.00
0.5	13.85	8.89	7.40	6.69
1	1.86	1.42	1.30	1.25
1.5	1.09	1.03	1.02	1.01
2	1.01	1.00	1.00	1.00

VI. An Illustrative Example

In this section, the application of the DS $\bar{X}$ chart under measurement errors is shown through an illustrative example. Considering a solar wafer manufacturing process where the interested quality characteristic Y is the maximum open-circuit voltage (unit in volts). After a long time of study (Phase I) based on a large database, the values of μ_0, σ_0^2 and σ_ε^2 are given as 0.52, 0.02 and 0.01, respectively yielding $\gamma^2 = \sigma_\varepsilon^2 / \sigma_0^2$ = 0.01/0.02 = 0.5. In this example, it is assumed that ARL_0 = 370, $\bar{n}$ = 5, δ_{opt} = 1.0. The design parameters of the DS $\bar{X}$ chart given by [12] are n_1 = 4, n_2 = 10, w = 1.63837, k_1 = 3.20638, and k_2 = 3.003. The limits at the first stage of the DS scheme (i.e. $-k_1$, $-w$, w, k_1) are based on the left scale, while the limits at the second stage of the DS scheme (i.e. $-k_2$ and k_2) are based on the right scale. For the linearly covariate error model, A = 0, B = 1 and m = 1.

The plotted samples without and with measurement errors are represented graphically in Figures 2(a) and 2(b), respectively. Note that in these figures, the solid and hollow dots represent the plotted Z_i at the first and second stages of the DS scheme, respectively. The values of Z_i of the first stage of the DS scheme are plotted using the left scale, where I_1 = [−1.63837, 1.63837]; I_2 = [−3.20638, −1.63837] ∪ [1.63837, 3.20638]; and I_4 = (−∞, −3.20638) ∪ (3.20638, ∞); while the values of Z_i at the second stage of the DS scheme are plotted using the right scale, where I_3 = [−3.003, 3.003] and I_5 = (−∞, −3.003) ∪ (3.003, ∞).

As it can be noticed from Figure 2(a), the sample at sampling point 12 falls outside the limits, indicating an out-of-control signal is detected at this sample since Z_i = 2.44 $\in I_2$ at the first stage of the DS scheme and Z_i = 3.58 $\in I_5$ at the second stage of the DS scheme.

In Figure 2(b), all the samples from sampling point i = 1 to i = 20 are plotted within the limits, indicating the process is in in-control state. Note that the out-of-control signal is not triggered at sampling point 12 since $Z_i = 2.00 \in I_2$ at the first stage of the DS scheme and $Z_i = 2.60 \in I_3$ at the second stage of the DS scheme.

By comparing Figure 2(a) and Figure 2(b), the out-of-control signal is detected at one sample (i.e. sample 12) for the case without measurement errors (see Figure 2(a)), whereas there is no out-of-control signal when measurement errors exist (see Figure 2(b)). This shows that in the presence of measurement errors, the DS $\bar{X}$ chart is slower in detecting shift in the process mean compared to the case without measurement errors.

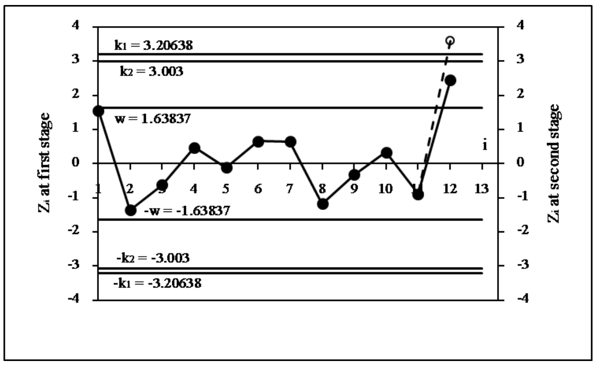

Figure 2(a): DS $\bar{X}$ chart for the illustrative example (without measurement error)

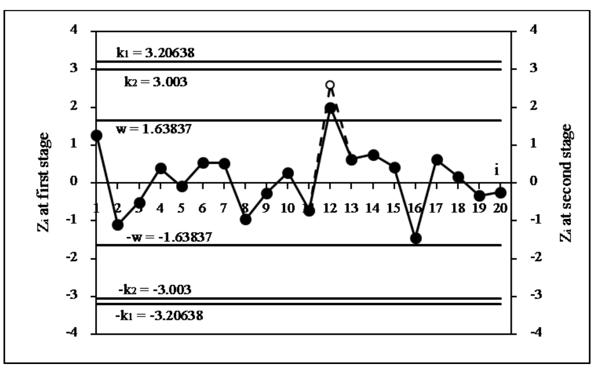

Figure 2(b): DS $\bar{X}$ chart for the illustrative example (with measurement error)

VII. CONCLUSIONS

The effect of measurement errors on the performance of the DS $\bar{X}$ chart is investigated through numerical studies. The linearly covariate error model in [1] is used. The numerical results show the deterioration in the performance of the DS $\bar{X}$ chart in the presence of measurement errors, having a negative effect on the chart's performance. The negative effect of measurement errors on the DS $\bar{X}$ chart can be reduced by using multiple measurements. This negative effect can also be reduced by increasing the *B* value of the linearly covariate error model.

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IX. APPENDIX

Let $\bar{Y} = (n_1\bar{X}_1 + n_2\bar{X}_2)/(n_1+n_2)$ be the sample mean at the second stage of the DS scheme, then the probability for $\bar{Y}$ to be plotted within Region I_3 is

$$\Pr(\bar{Y} \in I_3)$$
$$= \Pr(\mathrm{LCL}_2 < \bar{Y} < \mathrm{UCL}_2)$$
$$= \Pr\left(\mathrm{LCL}_2 < \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2} < \mathrm{UCL}_2\right)$$
$$= \Pr\left(A + B\mu_0 - k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)} < \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2} < A + B\mu_0 + k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)}\right)$$
$$= \Pr\left(A + B\mu_0 - k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)} < \frac{n_1(\bar{X}_1 - A - B\mu) + n_2(\bar{X}_2 - A - B\mu)}{n_1+n_2} + A + B\mu < A + B\mu_0 + k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)}\right).$$

Given that $Z_1 = \bar{X}_1 - (A+B\mu)/\sqrt{\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)/n_1}$ and $Z_2 = \bar{X}_2 - (A+B\mu)/\sqrt{\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)/n_2}$, then

$$\Pr(\bar{Y} \in I_3)$$
$$= \Pr\left(-k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)} < \frac{n_1\left(Z_1\sqrt{\frac{1}{n_1}\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)}\right)}{n_1+n_2} + \frac{n_2\left(Z_2\sqrt{\frac{1}{n_2}\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)}\right)}{n_1+n_2} - B\delta\sigma_0 < k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2\sigma_0^2 + \sigma_\varepsilon^2/m\right)}\right),$$

where δ = $(\mu_0-\mu)/\sigma_0$,. Dividing the numerator and the denominator by σ_0, we have

$$\Pr(\bar{Y} \in I_3)$$
$$= \Pr\left(-k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2 + \frac{\sigma_\varepsilon^2}{m\sigma_0^2}\right)} < \frac{\left(Z_1\sqrt{n_1\left(B^2 + \frac{\sigma_\varepsilon^2}{m\sigma_0^2}\right)}\right)}{n_1+n_2} + \frac{\left(Z_2\sqrt{n_2\left(B^2 + \frac{\sigma_\varepsilon^2}{m\sigma_0^2}\right)}\right)}{n_1+n_2} - B\delta < k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2 + \frac{\sigma_\varepsilon^2}{m\sigma_0^2}\right)}\right).$$

Given that $\frac{\sigma_\varepsilon^2}{\sigma_0^2} = \gamma^2$, then

$$\Pr(\bar{Y} \in I_3)$$
$$= \Pr\left(-k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2 + \gamma^2/m\right)} < \frac{\left(Z_1\sqrt{n_1\left(B^2+\gamma^2/m\right)}\right)}{n_1+n_2} + \frac{\left(Z_2\sqrt{n_2\left(B^2+\gamma^2/m\right)}\right)}{n_1+n_2} - B\delta < k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2+\gamma^2/m\right)}\right)$$
$$= \Pr\left(-k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2 + \gamma^2/m\right)} + B\delta < \frac{\left(Z_1\sqrt{n_1\left(B^2+\gamma^2/m\right)}\right)}{n_1+n_2} + \frac{\left(Z_2\sqrt{n_2\left(B^2+\gamma^2/m\right)}\right)}{n_1+n_2} < k_2\sqrt{\frac{1}{n_1+n_2}\left(B^2+\gamma^2/m\right)} + B\delta\right).$$

Dividing the numerator and denominator by $\sqrt{B^2+\gamma^2/m}$ obtains

$$\Pr(\bar{Y} \in I_3) = \Pr\left(-k_2\sqrt{\frac{1}{n_1+n_2}} + \frac{B\delta}{\sqrt{B^2+\gamma^2/m}} < \frac{Z_1\sqrt{n_1}}{n_1+n_2} + \frac{Z_2\sqrt{n_2}}{n_1+n_2} < k_2\sqrt{\frac{1}{n_1+n_2}} + \frac{B\delta}{\sqrt{B^2+\gamma^2/m}}\right).$$

Multiplying the numerator and denominator with (n_1+n_2) gives

$$\Pr(\bar{Y} \in I_3) = \Pr\left(-k_2\sqrt{n_1+n_2} + \frac{B\delta(n_1+n_2)}{\sqrt{\left(B^2+\gamma^2/m\right)}} < Z_1\sqrt{n_1} + Z_2\sqrt{n_2} < k_2\sqrt{n_1+n_2} + \frac{B\delta(n_1+n_2)}{\sqrt{\left(B^2+\gamma^2/m\right)}}\right).$$

Dividing the numerator and denominator by $\sqrt{n_2}$, we have

$$\Pr(\bar{Y} \in I_3)$$
$$= \Pr\left(-k_2\sqrt{\frac{n_1+n_2}{n_2}} + \frac{B\delta(n_1+n_2)}{\sqrt{n_2\left(B^2+\gamma^2/m\right)}} < Z_1\sqrt{\frac{n_1}{n_2}} + Z_2 < k_2\sqrt{\frac{n_1+n_2}{n_2}} + \frac{B\delta(n_1+n_2)}{\sqrt{n_2\left(B^2+\gamma^2/m\right)}}\right)$$
$$= \Pr\left(-k_2\sqrt{\frac{n_1+n_2}{n_2}} + \frac{B\delta(n_1+n_2)}{\sqrt{n_2\left(B^2+\gamma^2/m\right)}} - Z_1\sqrt{\frac{n_1}{n_2}} < Z_2 < k_2\sqrt{\frac{n_1+n_2}{n_2}} + \frac{B\delta(n_1+n_2)}{\sqrt{n_2\left(B^2+\gamma^2/m\right)}} - Z_1\sqrt{\frac{n_1}{n_2}}\right)$$
$$= \Phi\left(k_2\sqrt{\frac{n_1+n_2}{n_2}} + \frac{B\delta(n_1+n_2)}{\sqrt{n_2\left(B^2+\gamma^2/m\right)}} - Z_1\sqrt{\frac{n_1}{n_2}}\right) - \Phi\left(-k_2\sqrt{\frac{n_1+n_2}{n_2}} + \frac{B\delta(n_1+n_2)}{\sqrt{n_2\left(B^2+\gamma^2/m\right)}} - Z_1\sqrt{\frac{n_1}{n_2}}\right).$$

Given that p_{a2} is the probability that a sample is plotted within Region I_2 at the first stage of the DS scheme and the sample is then plotted within Region I_3 at the second stages of the DS scheme, then

$$p_{a2} = \Pr\left(\bar{Y} \in I_3 \text{ and } \bar{X}_1 \in I_2^*\right)$$
$$= \int_{I_2^*} \Pr\left(\bar{Y} \in I_3 \middle| \bar{X}_1 = x\right) f(x)\,dx,$$

where $I_2^* \in$ [LCL$_1$, LWL] $\cup$ [UWL, UCL$_1$]. Since the standardized value of $\overline{X}_1$ can be used to monitor the process, then

$$p_{a2} = \int_{I_2} \Pr\left(\overline{Y} \in I_3 \middle| Z_1 = z\right) f(z)dz$$

$$= \int_{I_2} \left(\Phi\left(k_2 \sqrt{\frac{n_1 + n_2}{n_2}} + \frac{B\delta(n_1 + n_2)}{\sqrt{n_2\left(B^2 + \gamma^2/m\right)}} - z\sqrt{\frac{n_1}{n_2}} \right) - \Phi\left(-k_2 \sqrt{\frac{n_1 + n_2}{n_2}} + \frac{B\delta(n_1 + n_2)}{\sqrt{n_2\left(B^2 + \gamma^2/m\right)}} - z\sqrt{\frac{n_1}{n_2}} \right)\right)\phi(z)dz,$$

where $I_2 \in [-k_1, -w] \cup [w, k_1]$; and $\phi(z)$ is the probability density function of the standardized normal random variable.