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MODELING THE PRICE OF HYBRID EQUITY WARRANTS UNDER STOCHASTIC VOLATILITY AND INTEREST RATE

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Abstract: Previous studies revealed that most local researchers frequently used the Black Scholes model to price equity warrants. However, the Black Scholes model was perceived of possessing too many drawbacks, such as big errors of estimation and mispricing of equity warrants. In this work, we consider the problem of pricing hybrid equity warrants based on a hybrid model of stochastic volatility and stochastic interest rate. The integration of stochastic interest rate using the Cox-Ingersoll-Ross (CIR) model, along with stochastic volatility of the Heston model was first developed as a hybrid model. We solved the governing stochastic equations and come up with analytical pricing formulas for hybrid equity warrants. This provides an alternative method for valuation of equity warrants, compared to the usual practice of utilizing the Black Scholes pricing formula.

Keywords: Equity warrants; stochastic; Cox-Ingersoll-Ross model; Heston model; hybrid models.

I. INTRODUCTION

Warrant is one of the financial instruments in Bursa Malaysia. It gives the holder the right to buy the underlying asset at a specified price and specified time. In Malaysia, the warrant market holds a great importance in economic industries. According to [1], the general formula of equity warrant is given by

$$W(T) = \frac{1}{N + Mk}(kS(T) - NG)^{+},$$

where W is the valuation of the equity warrant, N represents the number of shares for common stocks, M represents the number of shares for equity warrants outstanding, and S represents the current value of underlying asset. At the occurrence of payment denoted by G, each warrant entitles the warrant holder to earn k shares at time T.

Since the definition of warrants and option are corresponding to each other, it is natural to evaluate equity warrants using call option pricing models such as Black Scholes, since the model is one of the famous models utilized in pricing options in most markets, including Malaysia as discussed in [2]. Despite that**,** it was found that the prices for the Black Scholes model tend to under-price the equity warrants. Research conducted by [2] revealed that the pricing for more than 25, 000 daily warrants using the Black Scholes model ended with big errors of estimation compared to other models tested. Examples of research works involving Black Scholes model to price warrants and options can be referred from [3, 4, 5]. As reported by[3], the Black-Scholes model had led to the mispricing of call warrants, in terms of under or overvaluation of the price. In addition, [4] stated that in regards to pricing options in Malaysia market using Black-Scholes model, it was found that the model was not reliably precise and accurate. MAPE of the Black- Scholes model indicated 78.5% errors, in contrast to another model, 4F (Four Factor) option pricing model which had MAPE of 46.8%. Also, [5] found that the diluted-adjusted Black Scholes (DABS) model was more accurate in pricing warrants, compared to the Black Scholes model based on the examination of the highest error.

Pursuant to [6], the resolution of the financial framework, irrespective portfolio allocation, assets price, or risk management depends on simulation of a discriminative version of stochastic differential equations (SDEs). The author further reported the best way to encounter SDEs is to discretize them and apply Monte Carlo simulation in numerical situations. Moreover, the Euler method is commonly used in order to discretize SDEs. However, in an informative paper, [7] illustrated the compromise between the discretization error and the Monte Carlo averaging error. The asymptotic error distribution has been discovered by [8] and [9] for the Euler discretization method.

Besides that, amongst the variables applied in pricing financial derivatives is the short-term interest rate, where the short rate is varying over time. Accordingly, [2] discovered that the short-term interest rate of the Black Scholes model is known and constant through time, which may be troublesome to the long life of an equity warrant. Hence, it is significant to consider the circumstances of pricing equity warrants under stochastic interest rate to allow the fluctuations in the price of the underlying asset. Apart from that, previous studies were concerned that the Black Scholes model was not representative of the real world based on several assumptions. In fact, the assumption of constant volatility by the previous authors may cause significant mispricing when employed to evaluate exotic features. Without doubt, constant volatility is contrary to the real market, since it is only appropriate over short term maturity, but never constant in a long-term period. Thus, it is crucial to use stochastic volatility model when pricing warrants. Furthermore,[3] highlighted that the warrant was overpriced when the volatility was too high, and was under-priced when the volatility was too low. Nevertheless, for the case of pricing warrant, there is a gap left in the literature regarding application of stochastic volatility [10].

In conformity with our present knowledge, the analysis for pricing equity warrants, which establishes hybrid models of stochastic interest rate and stochastic volatility, has not been implemented yet [11,12]. Research regarding pricing formulation for hybrid variance swaps under these stochastic dynamics had been conducted by [13]. Thus, the novelty of this work is to develop an analytic pricing formula for equity warrants by incorporating stochastic interest rate from the Cox-Ingersoll-Ross (CIR) model, together with stochastic volatility from the Heston model. Thereby, besides reducing inaccuracies caused from constant interest rate and constant volatility, we can also provide a preferable market characterization through hybridization of stochastic interest rate and stochastic volatility.

II. MODELING TECHNIQUES

A. Pricing Model for Hybrid Equity Warrants

We set $S(t)$ as the asset price governed by μ, which is the drift, and $v(t)$ as its volatility. In addition, $v(t)$ is the instantaneous variance process with mean-reverting parameter k, long term mean of θ and σ as its volatility. We define $r(t)$ as the instantaneous interest rate, with α as the speed of the mean reversion, β as the interest rate term and η controls the volatility. Consider the following general Heston – CIR model:

$$dS(t) = \mu S(t)dt + \sqrt{v(t)}S(t)\,dw_1(t),$$
$$dv(t) = k\big(\theta - v(t)\big)dt + \sigma\sqrt{v(t)}\,dw_2(t),$$
$$dr(t) = \alpha\big(\beta - r(t)\big)dt + \eta\sqrt{r(t)}\,dw_3(t). \qquad (1)$$

Here,$\big(dw_1(t), dw_2(t)\big) = \rho dt$,$\big(dw_1(t), dw_3(t)\big) = 0$, $\big(dw_2(t), dw_3(t)\big) = 0$, and the correlation is denoted by constant ρ with $-1 \le \rho \le 1$. Also, $2k\theta \ge \sigma^2$ and $2\alpha\beta \ge \eta^2$ respectively.

Therefore, under the risk-neutral probability measure$\mathbb{Q}$,the above equation can be transformed into the following system of differential equations:

$$\begin{cases} dS(t) = r(t)S(t)\,dt + \sqrt{v(t)}\,S(t)\,d\widetilde{w_1}(t), \\ dv(t) = k^*\big(\theta^* - v(t)\big)\,dt + \sigma\sqrt{v(t)}\,d\widetilde{w_2}(t), \\ dr(t) = \alpha^*\big(\beta^* - r(t)\big)\,dt + \eta\sqrt{r(t)}\,d\widetilde{w_3}(t), \end{cases} \qquad (2)$$

with $k^* = k + \lambda_1, \theta^* = \frac{k\theta}{k+\lambda_1}, \alpha^* = \alpha + \lambda_2;\ \beta^* = \frac{\alpha\beta}{\alpha+\lambda_2}$ as the risk-neutral parameters; and $\widetilde{w}i(t)(1 \le i \le 3)$ is defined as the Brownian motions under $\mathbb{Q}$. λ_j $(j = 1,2)$ is the premium of volatility or interest rate risk.

The following system under the forward measure $\mathbb{Q}^T$can be obtained by applying the Cholesky decomposition of a correlation matrix and the Radon-Nikodym derivative such that:

$$\begin{bmatrix} \frac{ds(t)}{s(t)} \\ dv(t) \\ dr(t) \end{bmatrix} = \begin{bmatrix} r(t) \\ k^*\big(\theta^* - v(t)\big) \\ \alpha^*\beta^* - [\alpha^* + B(t,T)\eta^2]r(t) \end{bmatrix} dt + \Sigma \times \mathrm{C}$$

$$\times \begin{bmatrix} dw_1^*(t) \\ dw_2^*(t) \\ dw_3^*(t) \end{bmatrix},$$

Where

$$\Sigma = \begin{bmatrix} \sqrt{v(t)} & 0 & 0 \\ 0 & \sigma\sqrt{v(t)} & 0 \\ 0 & 0 & \eta\sqrt{r(t)} \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 \\ \rho & \sqrt{1-\rho^2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$B(t,T) = \frac{2\left(e^{(T-t)\sqrt{(\alpha^*)^2+2\eta^2}}-1\right)}{2\sqrt{(\alpha^*)^2+2\eta^2}+\alpha^*+\sqrt{(\alpha^*)^2+2\eta^2}\left(e^{(T-t)\sqrt{(\alpha^*)^2+2\eta^2}}-1\right)},$$

$$CC^T = \begin{bmatrix} 1 & \rho & 0 \\ \rho & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{and} \begin{bmatrix} d\widetilde{w_1}(t) \\ d\widetilde{w_2}(t) \\ d\widetilde{w_3}(t) \end{bmatrix} = C \times \begin{bmatrix} dw_1^*(t) \\ dw_2^*(t) \\ dw_3^*(t) \end{bmatrix} (3)$$

B. The Evaluation of Hybrid Equity Warrants

Define $W(t)$ as the function of the current value of the underlying asset $S(t)$, the stochastic interest rate $r(t)$, the stochastic volatility $v(t)$ and time t, given by:

$W(t) = W(S(t), v(t), r(t), t)$.

The theorem of Feynman Kac provides us

$$\frac{\partial W}{\partial t} + \frac{1}{2}vS^2\frac{\partial^2 W}{\partial s^2} + \frac{1}{2}\sigma^2 v\frac{\partial^2 W}{\partial v^2} + \frac{1}{2}\eta^2 r\frac{\partial^2 W}{\partial r^2} + rs\frac{\partial W}{\partial s} + k^*(\theta^* - v)\frac{\partial W}{\partial v} + \alpha^*\beta^* - (\alpha^* + B(t,T)\eta^2)r\frac{\partial W}{\partial r} + \rho\sigma vS\frac{\partial^2 W}{\partial W\partial v} = 0 \quad (4)$$

with boundary condition

$W(T) = \frac{1}{N+Mk}(kS(T) - NG)^+$.(5)

Proposition 1:

When the underlying asset has the dynamics (3) and the payoff function follows $W(S, v, r, T) = H(s)$ at expiry T, then the PDE system solution for the derivative value is associated as:

$$\begin{cases} \frac{\partial W}{\partial t} + \frac{1}{2}vs^2\frac{\partial^2 W}{\partial s^2} + \frac{1}{2}\sigma^2 v\frac{\partial^2 W}{\partial v^2} + \frac{1}{2}\eta^2 r\frac{\partial^2 W}{\delta r^2} + rs\frac{\partial W}{\partial s} \\ +k^*(\theta^* - v)\frac{\partial W}{\partial v} + \alpha^*\beta^* - (\alpha^*\, B(t,T)\eta^2)r\frac{\partial W}{\partial r} \; (6) \\ +p\sigma vs\frac{\partial^2 W}{\partial W\partial v}, W(S, v, r, T) = H(s) \end{cases}$$

and can be written in semi-closed formula as follows:

$W(x, v, r, \tau) = \mathcal{F}^{-1}\left[e^{C(\omega,\tau)+D(\omega,\tau)v+E(\omega,\tau)r}\mathcal{F}[H(e^x)]\right]$,

where $x = \ln S, \tau = T - t, i = \sqrt{-1}$, and ω is the variable of Fourier transform such that:

$$\begin{cases} D(\omega,\tau) = \frac{a+b}{\sigma^2}\cdot\frac{1-e^{b\tau}}{1-ge^{b\tau}}, \\ a = k^* - \rho\sigma\omega i,\; b = \sqrt{a^2 + \sigma^2(\omega^2 + \omega i)}, \\ g = \frac{a+b}{a-b}, \end{cases} \quad (7)$$

$E(\omega, \tau)$ and $C(\omega, \tau)$ satisfy the following ODE system

$$\begin{cases} \frac{dE}{d\tau} = \frac{1}{2}\eta^2 E^2 - (\alpha^* + B(T-\tau, T)\eta^2)E + \omega i, \\ \frac{dC}{d\tau} = k^*\theta^* D + \alpha^*\beta^* E, \end{cases} \quad (8)$$

with the initial conditions $C(\omega, 0) = 0$ and $E(\omega, 0) = 0$.

C. The Payoff Function

Following Proposition 1 and our terminal condition, we can state $H(S) = kS(T) - NG$, and the inverse Fourier transform could be specifically developed. The Fourier generalized transformation can be defined as:

$$\mathcal{F}\left[e^{i\xi x}\right] = 2\pi\delta_\xi(\omega), \quad (9)$$

where ξ is any complex number and $\delta_\xi(\omega)$ is the generalized delta function satisfying

$\int_{-\infty}^{\infty} \delta_\xi(\omega)\, \Phi(x)dx = \Phi(\xi)$.(10)

Letting $x = ln\, S$, we focus specifically on the payoff function by implementing the generalized Fourier transform, given by $\mathcal{F}[ke^X - NG]$. Here, $\mathcal{F}$ is the Fourier transform; along with *k*, *N* and *G* which are all constants.

Using the property $e^{iw_0 t} = 2\pi\delta(\omega - \omega_0)$, we obtain the following

$\mathcal{F}[ke^X - NG]$

$= NG\mathcal{F}\left[\frac{ke^X}{NG}\right] - NG\mathcal{F}[1]$.

Since $\mathcal{F}\left[\frac{e^X}{NG}.k\right] = \frac{2k\pi\,\delta_{-i}(\omega)}{NG}$ and $\mathcal{F}[1] = 2\pi\delta_0(w)$,

then

$$\mathcal{F}NG\left[\frac{ke^X}{NG} - 1\right] = \left(\frac{2\pi k\delta_{-i}(\omega)}{NG} - 2\pi\delta_0(\omega)\right)NG$$

$$= NG.\,2\pi\left[\frac{k\delta_{-i}(\omega)}{NG} - \delta_0(\omega)\right]. \quad (11)$$

Moving on, we provide the solution to (6) as

$W(S, v, r, \tau)$

$$= \mathcal{F}^{-1}\left[e^{C(\omega,\tau)+D(\omega,\tau)v+E(\omega,\tau)r}.\,NG\; 2\pi\left[\frac{k\delta_{-i}(\omega)}{NG} - \delta_0(\omega)\right]\right]$$

$$= \int_{-\infty}^{\infty} e^{C(\omega,\tau)D(\omega,\tau)v+E(\omega,\tau)r} . NG \left[\frac{k\delta_{-i}(\omega)}{NG} - \delta_0(\omega) \right] e^{x\omega i}\, d\omega$$

$$= NG \left[\frac{ke^{C(\omega,\tau)+D(\omega,\tau)v+E(\omega,\tau)r+x\omega_i}}{NG} \Big|_{\omega=-i} - e^{C(\omega,\tau)+D(\omega,\tau)v+E(\omega,\tau)r+x\omega_i} \Big|_{\omega=0} \right]$$

$$= NG \left[\frac{k}{NG} e^{\tilde{C}(\tau)+\tilde{D}(\tau)v+\tilde{E}(\tau)r} - 1 \right] \quad (12)$$

where $\tilde{C}(\tau), \tilde{D}(\tau)$ and $\tilde{E}(\tau)$ are the representatives for $C(-i,\tau), D(-i,\tau)$ and $E(-i,\tau)$ respectively.

Finally, we give the final forms for $\tilde{C}(\tau), \tilde{D}(\tau)$ and $\tilde{E}(\tau)$ as follows

$$\begin{cases} \tilde{D}(\tau) = \frac{\tilde{a}+\tilde{b}}{\sigma^2} . \frac{1-e^{\tilde{b}\tau}}{1-\tilde{g}e^{b\tau}}, \tilde{a} = k^* - \rho\sigma, \\ \tilde{g} = \frac{\tilde{a}+\tilde{b}}{\tilde{a}-\tilde{b}}, \tilde{b} = \sqrt{\tilde{a}^2} \end{cases} \quad (13)$$

along with $\tilde{E}(\tau)$ and $\tilde{C}(\tau)$ satisfying the following ODE system

$$\begin{cases} \frac{d\tilde{E}}{d\tau} = \frac{1}{2}\eta^2\tilde{E}^2 - (\alpha^* + B(T-\tau,T)\eta^2)\tilde{E} + 1, \\ \frac{d\tilde{C}}{d\tau} = k^*\theta^*\tilde{D} + \alpha^*\beta^*\tilde{E}. \end{cases}$$

III. CONCLUSION

This paper investigates the pricing of hybrid equity warrants by considering the framework of stochastic interest rate and stochastic volatility. We solved the governing PDE in the model and derived an analytical pricing formula for hybrid equity warrants based on the Heston–CIR hybrid model.

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